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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS. VOLUME I. STRUCTURAL SYSTEM IDENTIFICATION FROM MULTIPOINT EXCITATION

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Kaman Aerospace Corporation

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VOLUME I
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By

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Alex Berman
Nicholas Giansante

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EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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KAMAN AEROSPACE CORPORATION
BLOOMFIELD, CONNECTICUT

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This report contains the theoretical derivation and the presentation of a methodology for system identification of structures. Computer experiments were run to verify this methodology.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. Arthur J. Gustafson, Technology Applications Division.

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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS

Volume I
Structural System Identification From Multipoint Excitation

Final Report

Kaman Report R-1001-1

By

William G. Flannelly
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Prepared by

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13. ABSTRACT

The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data and the approximate natural frequency associated with each mode. The natural frequencies are readily available from response plots. Thus, using only impedance-type test data without the use of an intuitive mathematical model, the equations of motion for the structure may be obtained -- a process referred to as system identification.

In conjunction with the determination of the aforementioned parameters, the eigenvector or mode shape and generalized mass corresponding to each natural frequency are also calculated.

A digital computer program was generated to numerically test the system identification theory. Computer experiments were conducted to test the sensitivity of the theory to errors in input data.

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computer simulations						
computer experiments						
error sensitivity						
simulated test data						
computer programs						

FOREWORD

The work presented in this report was performed by Kaman Aerospace Corporation under Contract DAAJ02-70-C-0012 (Task 1F162204AA4301) for the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia. The program was implemented under the technical direction of Mr. Joseph H. McGarvey of the Reliability and Maintainability Division* and Mr. Arthur J. Gustafson of the Structures Division.** The report is presented in four volumes, each describing a separate phase of the basic theory of structural dynamic testing using impedance techniques.

Volume I presents the results of an analytical and numerical investigation of the practicality of system identification using fewer measurement points than there are degrees of freedom. The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data. Volume II describes the method of system identification wherein the necessary impedance data are experimentally determined by applying a force excitation at a single point on the structure. Volume III presents a method of determining the free-body dynamic responses from data obtained on a constrained structure. Volume IV describes a method of obtaining the equations for the combination of measured mobility matrices of a helicopter and its subsystems. The response of the combination of a helicopter and its subsystems is determined from data based on the experimental results of the main system and subsystems separately.

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*Division name changed to Military Operations Technology Division.

**Division name changed to Technology Applications Division.

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$[c]$	the damping matrix
$[d]$	a damping matrix; $[d] = \omega[c]$; for damping forces which are proportional to displacement
$\{f\}$	vector of external forces acting along the generalized coordinates
$\{\tilde{f}\}$	force phasor, $\{f\} = \{\tilde{f}\}e^{i\omega t}$
g_i	the structural damping coefficient of the i-th mode
i or j	indices; imaginary operator ($i = \sqrt{-1}$)
$\mathcal{K}_i$	the generalized stiffness of the i-th mode
$[k]$	the stiffness matrix
$\mathcal{M}_i$	the generalized mass of the i-th mode
$[m]$	the mass matrix
N or n	the number of degrees of freedom in the structure
$\{\dot{y}\}$	vector of velocities of the generalized coordinates
$\{\tilde{\dot{y}}\}$	velocity phasor, $\{\dot{y}\} = \{\tilde{\dot{y}}\}e^{i\omega t}$
$[Y_{(\omega)}]$	matrix of mobilities at forcing frequency ω; $[Y_{(\omega)}] = [\partial\tilde{\dot{y}}_i/\partial f_j]_{(\omega)}$

LIST OF SYMBOLS (Continued)

$Y^*_{i(\omega)}$ generalized mobility of the i-th mode at forcing frequency ω

$[\ddot{Y}]$ matrix of acceleration mobilities

$[\dot{z}_{(\omega)}]$ matrix of impedances at forcing frequency ω; $[z_{(\omega)}] = [\partial\tilde{f}_i/\partial\tilde{\dot{y}}_j]_{(\omega)}$

$\dot{z}^*_{i(\omega)}$ generalized impedance of the i-th mode at forcing frequency ω

$\bar{z}^*_{i(\omega)}$ complex conjugate of the i-th mode generalized impedance at forcing frequency ω

$|\dot{z}^*_{i(\omega)}|$ absolute value of the i-th mode generalized impedance at forcing frequency ω

$\{\gamma\}_i$ the i-th column of $[\Gamma]$; the gamma vector of the i-th mode; a left-hand eigenvector of $[k]^{-1}[m]$

$[\Gamma]$ the left-hand eigenvectors of $[k]^{-1}[m]$; $[\Phi]^{-T}$

δ_i^j Kronecker's delta

$\{\phi\}_i$ the modal vector of the i-th mode

$[\Phi]$ matrix of modal vectors

ω forcing frequency

Ω_i the natural frequency of the i-th mode

LIST OF SYMBOLS (Continued)

SUPERSCRIPTS

R	the real part of a complex quantity
I	the imaginary part of a complex quantity
*	a generalized parameter associated with a particular mode
T	the transpose
-T	the inverse transpose

SUBSCRIPTS

(ω)	the forcing frequency at which the quantity was measured or calculated
k	forcing frequency
	A dot over a quantity indicates differentiation with respect to time

BRACKETS

[ ],( )	matrix
$\lceil\ \rfloor$	diagonal matrix
{ }	column or row vector

INTRODUCTION

The success of a helicopter structural design is highly dependent on the ability to predict and control the dynamic response of the fuselage and mechanical components. Conventionally, this involves the formulation of intuitively based equations of motion. Ideally, this process would reduce the physical structure to an analytical mathematical model which would predict accurately the dynamic response characteristics of the actual structure. Obviously, the creation of such an intuitive abstraction of a complicated real structure requires considerable expertise and inherently includes a high degree of uncertainty. Structural dynamic testing is required to substantiate the analytical results. The analysis is modified until successful correlation is obtained between the analytical predictions and the test results. Finally, the mathematical model can be used to incorporate changes to improve the structural integrity of the helicopter.

This report describes the theory of structural dynamic testing using impedance techniques as applied to a mathematical model having fewer degrees of freedom than the structure. Reference 1 describes the method of obtaining a model directly from test measurements for a hypothetical structure which has the same number of degrees of freedom as the mathematical model. In reality, the number of degrees of freedom of a physical structure is infinite; therefore, the usefulness of model identification, necessarily with a finite number of degrees of freedom, using impedance testing techniques depends on the ability to simulate the real structure with a small mathematical model.

The process of deriving the equations of motion from test data is referred to as system identification. The only input information required in this theory is measured mobility data and the approximate natural frequency of each mode. This information can be obtained from impedance testing of the actual structure over the frequency range of interest yielding the second order, structurally damped linear equations of motion.

System identification theories to be of any practical engineering significance must be functional with a reasonable degree of experimental error. In this report, a series of computer experiments incorporating experimental errors was documented. This report presents a modification and extension of the analysis derived in Reference 1 such that an identified model with a finite number of degrees of freedom simulates the actual structure wherein the number of degrees of freedom is infinite.

THEORY

DERIVATION

The equations of motion in matrix form of a linear system are, as shown in Reference 1,

$$[m]\{\ddot{y}\} + [c]\{\dot{y}\} + [k]\{y\} = \{f\} \qquad (1)$$

Assume a steady-state solution of the form

$$\{\dot{y}\} = \{\tilde{\dot{y}}\}e^{i\omega t} \text{ and } \{f\} = \{\tilde{f}\}e^{i\omega t}$$

Substitute these equations into Equation (1) to give

$$\left[\left([m]\omega - \frac{1}{\omega}[k]\right)i + [c]\right]\{\tilde{\dot{y}}\} = \{\tilde{f}\} \qquad (2)$$

or

$$(i[\dot{z}^{I}_{\omega}] + [\dot{z}^{R}_{\omega}])\{\tilde{\dot{y}}\} \equiv [\dot{z}_{\omega}]\{\tilde{\dot{y}}\} = \{\tilde{f}\}$$

where $\dot{z}_{ij(\omega)}$ is defined herein as the element velocity impedance measured at ω.

The element impedance can also be expressed as

$$\dot{z}_{ij(\omega)} = \partial\tilde{f}_i/\partial\tilde{\dot{y}}_j$$

If Equation (2) is premultiplied by $[\Phi]^{-T}[\Phi]^{T}$ and post-multiplied by $[\Phi][\Phi]^{-1}$ where $[\Phi]$ is the matrix of modal vectors, the result is

$$[\Phi]^{-T}\left[i([\Phi]^{T}[m][\Phi]\omega - \frac{1}{\omega}[\Phi]^{T}[k][\Phi]) + [\Phi]^{T}[c][\Phi]\right][\Phi]^{-1} = [\dot{z}_{(\omega)}] \qquad (3)$$

The diagonal generalized mass is expressed by

$$\lceil \mathcal{M} \rfloor = [\Phi]^{T}[m][\Phi] \qquad (4)$$

The diagonal generalized stiffness is given by

$$\lceil \mathcal{K} \rfloor = [\Phi]^{T}[k][\Phi] \qquad (5)$$

Assume that

$$\frac{1}{\omega}[g\kappa] = [\Phi]^T[c][\Phi] \qquad (6)$$

such as would be expected from structural damping in a lightly damped structure. Substituting Equations (4), (5) and (6) into Equation (3) yields

$$[\dot{z}_{(\omega)}] = [\Phi]^{-T}\left[j(\eta\omega - \frac{\kappa}{\omega}) + \frac{g\kappa}{\omega} \right][\Phi]^{-1} \qquad (7)$$

Define the i-th modal impedance as

$$\dot{z}^*_{i(\omega)} = j(\eta_i\omega - \frac{\kappa_i}{\omega}) + \frac{g_i\kappa_i}{\omega}$$

and substitute into Equation (7) to give

$$[\dot{z}_{(\omega)}] = [\Phi]^{-T}\left[\dot{z}^*_{(\omega)} \right][\Phi]^{-1} \qquad (8)$$

The elemental mobility at forcing frequency ω is defined as

$\dot{Y}_{ij(\omega)} \equiv \partial\tilde{\dot{y}}_i/\partial f_j$ and is equal to the ratio of the velocity

phasor along the coordinate i to the external force phasor along the coordinate j when no other forces are externally applied. The full mobility matrix is given by

$$[\dot{Y}_{(\omega)}] = [\partial\tilde{\dot{y}}/\partial\tilde{f}]_{(\omega)} = [\partial\tilde{f}/\partial\tilde{\dot{y}}]_{(\omega)}^{-1} \equiv [z_{(\omega)}]^{-1} \qquad (9)$$

Therefore, using Equation (8) it is seen that

$$[\dot{Y}_{(\omega)}] = [\Phi]\left[\frac{1}{\dot{z}^*_{(\omega)}} \right][\Phi]^T \equiv [\Phi]\left[\dot{Y}^*_{(\omega)} \right][\Phi]^T \qquad (10)$$

The modal mobility of the i-th mode measured at ω is

$$\dot{Y}^*_{i(\omega)} = \dot{Y}^{*R}_{i(\omega)} + i\dot{Y}^{*I}_{i(\omega)} = \frac{1}{\dot{z}^*_{i(\omega)}} = \frac{\bar{\dot{z}}^*_{i(\omega)}}{(\dot{z}^*_{i(\omega)})^2}$$

$$= \frac{\dot{z}^{*R}_{i(\omega)} - i\dot{z}^{*I}_{i(\omega)}}{(\dot{z}^{*R}_{i(\omega)})^2 + (\dot{z}^{*I}_{i(\omega)})^2} = \frac{\frac{g_i\kappa_i}{\omega} - i(\eta_i\omega - \frac{\kappa_i}{\omega})}{(\frac{g_i\kappa_i}{\omega})^2 + (\eta_i\omega - \frac{\kappa_i}{\omega})^2}$$

Dividing numerator and denominator of the previous equation by the generalized mass $\mathcal{M}_i$

$$\dot{Y}^*_{i(\omega)} = \frac{\dfrac{g_i \mathcal{K}_i}{\mathcal{M}_i \omega} - i\left(\omega - \dfrac{\mathcal{K}_i}{\mathcal{M}_i \omega}\right)}{\mathcal{M}_i \left(\dfrac{g_i \mathcal{K}_i}{\omega \mathcal{M}_i}\right)^2 + \mathcal{M}_i \left(\omega - \dfrac{\mathcal{K}_i}{\mathcal{M}_i \omega}\right)^2}$$

Substituting the natural frequency of the i-th mode

$$\Omega_i = \sqrt{\frac{\mathcal{K}_i}{\mathcal{M}_i}}$$

$$\dot{Y}^*_{i(\omega)} = \frac{\dfrac{g_i \Omega_i}{\omega} - i\left(\omega - \dfrac{\Omega_i}{\omega}\right)}{\mathcal{M}_i \left(\dfrac{g_i \Omega_i{}^2}{\omega}\right) + \mathcal{M}_i \left(\omega - \dfrac{\Omega_i{}^2}{\omega}\right)}$$

Separating this equation into the real and imaginary components yields

$$\dot{Y}^*_{i(\omega)} = \frac{1}{\omega \mathcal{M}_i}\left(\frac{\omega}{\Omega_i}\right)^2 \left[\frac{g_i}{g_i{}^2 + \left(\dfrac{\omega^2}{\Omega^2} - 1\right)^2} - i\,\frac{\left(\dfrac{\omega^2}{\Omega_i{}^2} - 1\right)}{g_i{}^2 + \left(\dfrac{\omega}{\Omega^2} - 1\right)^2}\right] \tag{11}$$

Finally, from Equation (10), the real mobility may be written as

$$[\dot{Y}^R_{(\omega)}] = [\Phi]\left\lceil \dot{Y}^{*R}_{(\omega)} \right\rfloor [\Phi]^T \tag{12}$$

Reference 1 indicated that because the real modal mobilities of modes far removed from the forcing frequency become negligible compared to adjacent modes, the real mobility matrix at any frequency is ordinarily affected only by modes in close proximity to the forcing frequency. The measured real mobility matrix at a particular frequency reflects the influence of only the most dominant modes in that frequency of measurement region. Therefore, it is unrealistic to use the real mobility matrix measured at any specific frequency to determine parameters other than those associated with neighboring modes.

Reference 1 also shows that the imaginary modal mobilities of modes associated with frequencies less than the forcing frequency asymptotically approach a constant. An imaginary mobility matrix contains the effect of all lower modes in proportion to, or greater than, the magnitudes of their generalized masses. Therefore, it is impractical to use imaginary mobility matrices to evaluate properties associated with natural frequencies far above the forcing frequency.

These characteristics of the modal mobility make it impossible to determine the system parameters from the n equations in n unknowns obtained from mobility matrices measured at any two forcing frequencies.

Even if the modal mobility were amenable to determination of the system parameters, the precision of measurement which would be required to do this for most systems is impossible to achieve. The modal approach derived below avoids this problem.

DERIVATION OF THE DOMINANT MODE EIGENVALUE PROBLEM

Equation (10) may be written

$$[\dot{Y}_{(\omega)}] = [\Phi]\left[\dot{Y}^*_{(\omega)}\right][\Phi]^T = \sum_{i=1}^{N} \dot{Y}^*_{i(\omega)}\{\phi\}_i\{\phi_i\}^T \tag{13}$$

where $\{\phi\}$ is a column in $[\Phi]$ and N is the order of the matrices. Define $[\Gamma] = [\Phi]^{-T}$, and Equation (8) may be written as

$$[\dot{Y}_{(\omega)}]^{-1} = [Z_{(\omega)}] = [\Gamma]\left[\dot{Z}^*_{(\omega)}\right][\Gamma]^T = \sum_{i=1}^{N} \frac{1}{Y^*_{i(\omega)}}\{\gamma\}_i\{\gamma\}_i^T \tag{14}$$

where $\{\gamma\}$ is a column in $[\Gamma]$.

Each matrix $\left[Y^*_{i(\omega)}\{\phi_i\}\{\phi\}_i^T\right]$ and $\left[\frac{1}{\dot{Y}^*_{i(\omega)}}\{\gamma\}\{\gamma_i\}^T\right]$ in Equations (13)

and (14) is of rank one, but the summation of as many of these successive modal matrices as the order N of the matrix is a nonsingular matrix.

Similarly,

$$[\dot{Y}^{R}_{(\omega)}] = \sum_{i=1}^{N} \dot{Y}^{*R}_{i(\omega)} \{\phi\}_i \{\phi\}_i^T$$

$$[\dot{Y}^{I}_{(\omega)}] = \sum_{i=1}^{N} \dot{Y}^{*I}_{i(\omega)} \{\phi\}_i \{\phi\}_i^T$$

$$[\dot{Y}^{R}_{(\omega)}]^{-1} = \sum_{i=1}^{N} \frac{1}{\dot{Y}^{*R}_{i(\omega)}} \{\gamma\}_i \{\gamma\}_i^T$$

$$[\dot{Y}^{I}_{(\omega)}]^{-1} = \sum_{i=1}^{N} \frac{1}{\dot{Y}^{*I}_{i(\omega)}} \{\gamma\}_i \{\gamma\}_i^T \tag{15}$$

The iteration procedure used to solve the eigenvalue problem in Reference 1 employed the imaginary part of a mobility matrix measured at a frequency just above the N-th natural frequency. The method used to solve the eigenvalue problem in the present report, which was found to give more accurate results, utilizes the sum of the real parts of the mobility matrices measured near each of the natural frequencies associated with the actual model. It has been indicated previously that a measured real mobility matrix reflects the influence of only the most dominant modes in the vicinity of the forcing frequency. Therefore, summation of a discrete set of the real mobility matrices measured at forcing frequencies near the corresponding natural frequencies should contain precisely the information relevant to the model normal modes.

The eigenvalue problem may be formulated as follows. Consider the summation of the real mobility matrices measured at a discrete set of frequencies near the first NR natural frequencies. Take the inverse of this matrix and premultiply by a real mobility matrix measured at any frequency ω_k.

$$[\dot{Y}^R_{(\omega_k)}][\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{i(\omega_j)}]^{-1}$$

$$= \sum_{i=1}^{NR} \dot{Y}^{*R}_{i(\omega_k)}\{\phi_i\}\{\phi\}^T_i(\sum_{i=1}^{NR}\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}\{\phi_i\}\{\phi_i\}^T)^{-1}$$

$$= [\Phi]\left[\diagdown \dot{Y}^{*R}_{i(\omega_k)} \diagdown\right][\Phi]^T([\Phi]\left[\diagdown \sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)} \diagdown\right][\Phi]^T)^{-1}$$

$$= [\Phi]\left[\diagdown \dot{Y}^{*R}_{i(\omega_k)} \diagdown\right][\Phi]^T[\Phi]^{-T}\left[\diagdown \sum_{j=1}^{NR} \frac{1}{\dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^{-1}$$

$$= [\Phi]\left[\diagdown \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^{-1} \tag{16}$$

If Equation (16) is postmultiplied by $\{\phi\}_i$, there results

$$\left[\dot{Y}^R_{(\omega_k)}\right]\left[\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{i(\omega_j)}\right]^{-1}\{\phi\}_i = [\Phi]\left[\diagdown \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^{-1}\{\phi_i\}$$

but $[\Phi]^{-1}\{\phi_i\}$ yields a column matrix comprised of zeroes except for a 1 in the i-th position. Finally,

$$[\Phi]\left[\diagdown \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^{-1}\{\phi\}_i = \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}}\{\phi_i\}$$

The eigenvalue problem is finally formulated as

$$[\dot{Y}^R_{(\omega_k)}]\left[\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{(\omega_j)}\right]^{-1}\{\phi\}_i = \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^{*R}_{i(\omega_j)}}\{\phi\}_i \tag{17}$$

If the order of multiplication is reversed in Equation (16), an eigenvalue problem is developed in which the eigenvector is the gamma vector of the i-th mode. Consider the same parameters as in Equation (16); only the order of multiplication of the matrices is changed.

$$\left[\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{i(\omega_j)}\right]^{-1}[\dot{Y}^R_{(\omega_k)}]$$

$$= \left(\sum_{i=1}^{NR}\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}\{\phi_i\}\{\phi_i\}^T\right)^{-1}\left(\sum_{i=1}^{NR} \dot{Y}^{*R}_{i(\omega_k)}\{\phi_i\}\{\phi_i\}^T\right)$$

$$= \left([\Phi]\left[\diagdown \sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)} \diagdown\right][\Phi]^T\right)^{-1}[\Phi]\left[\diagdown \dot{Y}^{*R}_{i(\omega_k)} \diagdown\right][\Phi]^T$$

$$= [\Phi]^{-T}\left[\diagdown \frac{1}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^{-1}[\Phi]\left[\diagdown \dot{Y}^{*R}_{i(\omega_k)} \diagdown\right][\Phi]^T$$

$$= [\Phi]^{-T}\left[\diagdown \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown\right][\Phi]^T \tag{18}$$

By definition, $[\Gamma] = [\Phi]^{-T}$ and $[\Gamma]^{-1} = [\Phi]^T$; substituting into Equation (18) yields

$$\left[\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{i(\omega_j)}\right][\dot{Y}^R_{(\omega_k)}] = [\Gamma]\begin{bmatrix} \diagdown \dfrac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum\limits_{j=1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}} \diagdown \end{bmatrix}[\Gamma]^{-1} \tag{19}$$

If Equation (19) is postmultiplied by $\{\gamma\}_i$, a column of $[\Gamma]$, and the same procedure is followed as was used in obtaining Equation (17), Equation (19) becomes

$$\left[\sum_{\omega_j=\Omega_1}^{\Omega_{NR}} \dot{Y}^R_{i(\omega_j)}\right]^{-1}[\dot{Y}^R_{(\omega_k)}]\{\gamma\}_i = \frac{\dot{Y}^{*R}_{i(\omega_k)}}{\sum\limits_{\omega_j=\Omega_1}^{NR} \dot{Y}^{*R}_{i(\omega_j)}}\{\gamma\}_i \tag{20}$$

which is an eigenvalue problem with the eigenvector equal to the gamma vector of the i-th mode.

IDENTIFICATION OF STRUCTURAL PARAMETERS

A successful identification procedure, using normal mode techniques, should separate the effect of each mode in a mathematical sense, regardless of the number of stations where mobility measurements are taken on the structure. If satisfactory normal mode separation required a certain minimum number of measurement stations greater than the number of degrees of freedom chosen for the model, the most that can be expected is an approximate model, possibly including optimization procedures designed to satisfy all system constraints. This situation is considered in detail in Reference 2 in which a mathematical model is derived from test data such that identification of the structure is obtained closest to any specified analytical approximation.

Satisfactory normal mode separation requires that the values of $\dot{Z}^{*R}_{(\omega_j)}$ and $\dot{Z}^{*I}_{(\omega_j)}$ be independent of the number of degrees of freedom of the model. The values of the generalized mass ($\mathcal{M}_i$), the corresponding identified natural frequency (Ω_i), and the generalized stiffness as defined below are then also independent of the number of measurement stations.

$$\eta_i = \frac{\omega_k \dot{Z}^{*I}_{i(\omega_k)} - \omega_j \dot{Z}^{*I}_{i(\omega_j)}}{(\omega_k^2 - \omega_j^2)} \tag{21}$$

and

$$\Omega_i^2 = \omega_j \omega_k \frac{\omega_j \dot{Z}^{*I}_{i(\omega_k)} - \omega_k \dot{Z}^{*I}_{i(\omega_j)}}{\omega_k Z^{*I}_{i(\omega_k)} - \omega_j Z^{*I}_{i(\omega_j)}} \tag{22}$$

$$\kappa_i = \Omega_i^2 \eta_i \tag{23}$$

The two forcing frequencies (ω_k) and (ω_j) are chosen in the vicinity of the corresponding natural frequency which is available from test data. The generalized impedance of the i-th mode at forcing frequency (ω) is obtained from the generalized mobility of the i-th mode at forcing frequency (ω). It follows from Equation (13) that the modal mobilities are given by

$$\left\lceil \dot{Y}^*_{(\omega)} \right\rfloor = [\Phi]^{-1} [\dot{Y}_{(\omega)}] [\Phi]^{-T} \tag{24}$$

$$= [\Gamma]^T [\dot{Y}_{(\omega)}][\Gamma]$$

and, therefore, the orthogonality condition for gamma vectors is

$$\{\gamma\}_i^T [\dot{Y}_{(\omega)}] \{\gamma\}_i = \dot{Y}^*_{i(\omega)} \delta^j_i$$

The modal impedance of the i-th mode at ω_j is

$$\dot{Z}^*_{i(\omega_j)} = \frac{\dot{Y}^*_{i(\omega_j)}}{|\dot{Y}^*_{i(\omega_j)}|^2} = \frac{\dot{Y}^{*R}_{i(\omega_j)} - i\dot{Y}^*_{i(\omega_j)}}{|\dot{Y}^*_{i(\omega_j)}|^2}$$

It follows that

$$Z^{*I}_{i(\omega_j)} = \frac{-Y^{*I}_{i(\omega_j)}}{|Y^{*}_{i(\omega_j)}|^2}$$

and

$$Z^{*R}_{i(\omega_j)} = \frac{Y^{*R}_{i(\omega_j)}}{|Y^{*}_{i(\omega_j)}|^2}$$

The damping coefficient for the i-th mode is most readily given by

$$g_i = \frac{\omega_j Z^{*R}_{i(\omega_g)}}{\kappa_i} \qquad (25)$$

which follows directly from Equation (7). The damping coefficient for the i-th mode may also be obtained by

$$g_i = \left(\frac{\omega_j^2}{\Omega_i^2} - 1\right) \frac{Z^{*R}_{i(\omega_i)}}{Z^{*I}_{i(\omega_j)}} \qquad (26)$$

Using a measurement of real mobility taken precisely at resonance, the damping coefficient may be calculated using Equation (11) as

$$g_i = \frac{1}{Y^{*R}_{i(\Omega_i)}\Omega_i \eta_i}$$

PARAMETERS OF THE MATHEMATICAL MODEL

The elements of the influence coefficient matrix, being a measure of displacement per unit force, are independent of the number of measurement stations defining the order of the matrix. Conversely, the elements of the stiffness and mass matrices assume different values as the number of degrees of freedom of the model is changed. The identification procedure used in Reference 1 calculates both stiffness and mass matrices by summing the effects of each consecutive mode and defining the incomplete matrices as the sum up to and including a particular mode. If the order of the model

degrees of freedom is changed from ND for the structure to NR for the model, the corresponding incomplete mass and stiffness matrices will not be directly comparable, on a modal basis, to the structure mass and stiffness matrices. It is more expedient to identify the influence coefficient matrix [c] and the inverse of the mass matrix $[\mathcal{M}]$. Premultiplying Equation (4) by $[\Phi]^{-T}$ and postmultiplying $[\Phi]^{-1}$ and taking the inverse of the resulting equation yields

$$[\mathcal{M}] = [m]^{-1} = \sum_{i=1}^{NR} \frac{1}{\mathfrak{m}_i} \{\phi_i\}\{\phi_i\}^T \tag{28}$$

If the same operations are performed on Equation (5), the result is

$$[c] = [K]^{-1} = \sum_{i=1}^{NR} \frac{1}{\Omega^2 \mathfrak{m}_i} \{\phi_i\}\{\phi_i\}^T \tag{29}$$

Set $[c] = \frac{1}{\omega}[d]$ and using Equation (6) there results

$$\frac{1}{\omega}\lceil g\mathcal{K}\rfloor = \frac{1}{\omega}[\Phi]^T[d][\Phi]$$

Solving for the damping matrix yields

$$[d] = \phi^{-T}\lceil g\mathcal{K}\rfloor\phi^{-1}$$

Substituting $[\Gamma] = [\phi]^{-T}$, $[\Gamma]^T = [\Phi]^{-1}$ and $\lceil\mathcal{K}\rfloor = \lceil\Omega^2\mathfrak{m}\rfloor$ into the previous equation gives

$$[d] = [\Gamma]\lceil g\Omega^2\mathfrak{m}\rfloor[\Gamma]^T$$

The damping matrix can be expressed as

$$[d] = \sum_{i=1}^{NR} g_i\Omega_i^2\mathfrak{m}_i\{\gamma_i\}\{\gamma_i\}^T \tag{30}$$

ITERATION PROCEDURE

The calculation of the modal parameters such as generalized mass, stiffness and the corresponding natural frequency requires the generalized impedance at a particular frequency for each mode under consideration. The modal impedance is a function of the generalized mobility for the same mode and forcing frequency. As indicated in Equation (24), the modal mobilities are dependent upon the matrix of gamma vectors and its transpose. The iteration process as originally formulated

in the present work involved iteration on the normal mode vectors with a subsequent inversion operation to determine the gamma vectors. This sequence introduced errors into the system, with the result that the gamma vectors did not resemble the associated gamma vectors obtained from the specimen representing the actual structure. The iterated normal mode vectors obtained from the mathematical model were extremely close, particularly at the lower modes, to the specimen, or exact, normal mode vectors. Nevertheless, any discrepancy between the model iterated modal vectors and the exact values, however small, was magnified in the inversion process, causing the gamma vectors to bear little resemblance to the specimen gamma vectors. Therefore, it was deemed advisable to iterate on the gamma vectors directly and disperse with the intermediate inversion operation.

To equalize the effect of each modal mobility in the matrix iteration Equation (20), several normalization procedures were incorporated into the method. First, each real mobility matrix was normalized on the largest element of the respective matrix. This procedure proved satisfactory except in some situations where the elements of the mobility matrices were approximately of the same magnitude but the largest elements differed in algebraic sign. This resulted in a cancellation effect among the real mobility matrices and an incorrect summation, thereby causing erroneous calculated gamma vectors. A modification to the normalization technique was applied whereby the real mobility matrices at each forcing frequency were divided by the absolute value of the largest element in the respective matrices. As a further refinement on the normalization procedure, the real mobility matrix at each forcing frequency was normalized on the root mean square associated with each respective matrix. Occasionally, these operations also caused problems in the final modal generalized mass and natural frequency calculations. For example, if a mobility matrix calculated at a particular frequency contained one element that dominated the matrix, normalization of the mobility matrix on this element would effectively submerge the influence of the matrix in the summation of the real modal mobilities. Again, the calculated modal parameters would obviously be incorrect. Similarly, if several elements of the mobility matrix measured at a specific forcing frequency were of greater magnitude than the remaining elements, the root mean square value would be affected and normalization by this value would yield a matrix wherein the elements were substantially reduced. Therefore, any such matrix would not be realistically represented in the summation of the real mobility matrices; consequently, the modal generalized mass and natural frequency would be incorrect.

Finally, each mobility matrix was multiplied by the respective forcing frequency yielding an acceleration mobility. These acceleration mobility matrices were substituted for the velocity mobilities appearing in Equation (17) and Equation (20) when iterating for the mode shapes and gamma vectors respectively. This technique was also plagued with similar difficulties that the aforementioned normalization procedures incurred. Fortunately, when the computer experiments were executed incorporating any of the previously discussed normalization methods, the conditions which yielded erroneous results were readily discernible. In these instances, the calculations for the modal generalized mass and natural frequency produced results which were obviously incorrect.
For the conditions which were recognized to be in error, the computer experiments were reevaluated substituting a different normalization option. Generally, the results obtained by altering a normalization procedure yielded modal parameters which were correct.

INTERPRETATION OF ELEMENTS IN THE REDUCED MASS MATRIX

In general, it may be expected that the algebraic sum of all the elements of a reduced mass matrix from system identification will approximate the gross weight of the aircraft. Due partly to restraints, the sum of the elements should not exactly equal the gross weight, because masses at elastic restraints do not act as if they were ungrounded. Masses at pinned joints to ground do not even figure in the mass matrix because they do not move.

Individual mass elements cannot be interpreted as reflecting lumped physical weights at their assigned locations. The elements of any reduced mass matrix represent the inertial, as opposed to elastic and damping, dynamic effects of the two (for off-diagonal) degrees of freedom with which they are associated in an actual system having many more degrees of freedom than the model. The off-diagonal terms in a reduced mass matrix will usually be large and sometimes negative. The matrix will usually be fully populated.

The identified mass and stiffness matrices can be used to draw a dynamic circuit of the helicopter and, if any one were interested, it would be possible to construct an actual spring-mass system (utilizing both positive and negative springs and moments of inertia) which would be an exact physical duplication of the identified model, element by element, and would have the same natural frequencies and modal eigenvectors as the helicopter; but it would not "look" like a helicopter. Neither negative spring rates nor negative off-diagonal masses are physically unrealizable; the former are used by

Lockheed in its control system and by all light-switch manufacturers, the latter are the essential part of the dynamic antiresonant vibration isolator.

The objective is not to identify a system which "looks" like a helicopter but one which "performs" like a helicopter under various dynamic loadings. The physical interpretation of the ij-th element of the identified mass matrix, for example, is that the helicopter will generally exhibit a partial derivative of a force at i with respect to a response at j which has an effective* mass component that is the ij-th element of the identified mass matrix (similarly for the stiffness and damping matrices).

It is immaterial in the identification whether there are as many points on the structure as there are degrees of freedom in the model, or if up to three degrees of freedom (in orthogonal directions) occur at any one point. It is important only that elements in the motion vector have the properties of generalized coordinates for the holonomic model considered. An identified reduced model in which some of the displacement elements represent the orthogonal Cartesian or polar coordinates of a given structural point would look much like an identified model of a similar system with parallel coordinates of separate points.

The impedance matrix, of which the mass and stiffness matrices are terms, of a mathematical model of a larger system is a function of the size of the model, and the terms must reflect this. It was found that frequency-independent mass, stiffness and damping matrices as described can accurately reflect the responses of a continuous structure over a finite spectrum by approximating a lambda matrix the inverse of which very closely approximates the mobility. The spectral mobility matrix, even of an order that equals the number of degrees of freedom in the structure, cannot be expressed as a lambda matrix.

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*Not to be confused with the formal definition of "Effective Mass" as

$$ME_{jki} \equiv \frac{\{\phi\}_i^T [m] \{\phi\}_i}{\phi_{ji}\phi_{ki}}$$

THE REDUCED MASS MATRIX

Consider the actual structure to consist of an infinite number of degrees of freedom of which R degrees of freedom are retained in the model. The mobility

$$\left[\begin{array}{c|c} [Y_{RR}] & [Y_{RE}] \\ \hline [Y_{ER}] & [Y_{EE}] \end{array}\right] = \left[\begin{array}{c|c} [Z_{RR}] & [Z_{RE}] \\ \hline [Z_{ER}] & [Z_{EE}] \end{array}\right]^{-1} = \left(\left[\begin{array}{c|c} [K_{RR}] & [K_{RE}] \\ \hline [K_{ER}] & [K_{EE}] \end{array}\right] - \omega^2 \left[\begin{array}{c|c} [M_{RR}] & 0 \\ \hline 0 & [M_{EE}] \end{array}\right]\right)^{-1} \tag{31}$$

The model impedance is defined as the inverse of the mobility matrix in the R degrees of freedom:

$$[Z_m] \equiv [Y_{RR}]^{-1} = [Z_{RR}] - [Z_{RE}][Z_{EE}]^{-1}[Z_{ER}] = [K_m] - \omega^2[M_m] \tag{32}$$

The stiffness of the model, $[K_m]$, is the inverse of the RxR influence coefficients:

$$[K_m] \equiv [C_{RR}]^{-1} = [K_{RR}] - [K_{RE}][K_{EE}]^{-1}[K_{ER}] \tag{33}$$

From Equation (31),

$$[Z_{RR}] = [K_{RR}] - \omega^2[M_{RR}]$$

$$[Z_{EE}] = [K_{EE}] - \omega^2[M_{EE}]$$

$$[Z_{RE}] = [K_{RE}]$$

$$[Z_{ER}] = [K_{ER}]$$

Substitute into Equation (32)

$$[Z_m] = [K_m] - \omega^2[M_m] = [K_{RR}] - \omega^2[M_{RR}] - [K_{RE}] \Big([K_{EE}]$$

$$- \omega^2[M_{EE}]\Big)^{-1}[K_{ER}] = [K_{RR}] - [K_{RE}][K_{EE}]^{-1}[K_{ER}]$$

$$- \omega^2[M_{RR}] - [K_{RE}] \Big([I] - \omega^2[K_{EE}]^{-1}[M_{EE}]\Big)^{-1}[K_{EE}]^{-1}[K_{ER}]$$

$$+ [K_{RE}][K_{EE}]^{-1}[K_{ER}]$$

Substitute Equation (33). Then

$$[M_m] = [M_{RR}] + \frac{1}{\omega^2} [K_{RE}] \Big[\Big([I] - \omega^2[K_{EE}]^{-1}[M_{EE}]\Big)^{-1}$$

$$- [I]\Big] [K_{EE}]^{-1}[K_{ER}] = [M_{RR}] + \frac{1}{\omega^2} [K_{RE}] \Big[\Big([I]$$

$$- \omega^2[K_{EE}]^{-1}[M_{EE}]\Big)^{-1} - \Big([I] - \omega^2[K_{EE}]^{-1}[M_{EE}]\Big) \Big([I]$$

$$- \omega^2[K_{EE}]^{-1}[M_{EE}]\Big)\Big]^{-1} [K_{EE}]^{-1}[K_{ER}]$$

$$[M_m] = [M_{RR}] + [K_{RE}][K_{EE}]^{-1}[M_{EE}] \Big([I]$$

$$- \omega^2[K_{EE}]^{-1}[M_{EE}]\Big)^{-1}[K_{EE}]^{-1}[K_{ER}] \tag{34}$$

Equation (34) is the "exact" RxR reduced mass matrix of a system with an infinite number of degrees of freedom. Note that $[M_m]$ is not diagonal and is a function of forcing frequency.

The frequency dependency of the "exact" reduced mass matrix simply reflects the fact that R linear differential equations with constant coefficients cannot contain enough information to <u>exactly</u> reflect the action of an infinite number of degrees of freedom over a spectrum containing R modes. The frequency dependency makes it impractical to use this in a linear engineering mathematical model.

The "Consistent Mass Matrix" (Reference 3), often used in finite-element dynamics work, is also based on a model stiffness matrix $[K_m]$ being the inverse of the RxR influence coefficient matrix:

$$[K_m] = [C_{RR}]^{-1} = [K_{RR}] - [K_{RE}][K_{EE}]^{-1}[K_{ER}].$$

The kinetic energy of the structure is set equal to the

kinetic energy of the model:

$$\begin{Bmatrix} \dot{y}_R \\ \dot{y}_E \end{Bmatrix}^T \left[\begin{array}{c|c} [M_{RR}] & 0 \\ \hline 0 & [M_{EE}] \end{array}\right] \begin{Bmatrix} \dot{y}_R \\ \dot{y}_E \end{Bmatrix} = \{\dot{y}_R\}^T [M_{RR}] \{\dot{y}_R\}$$

$$+ \{\dot{y}_E\}^T [M_{EE}] \{\dot{y}_E\} = \{\dot{y}_R\}^T [M_m] \{\dot{y}_R\} \tag{35}$$

It is implicitly assumed, however, that the inertial forces occur only along the R generalized coordinates, giving

$$\left[\begin{array}{c|c} [K_{RR}] & [K_{RE}] \\ \hline [K_{ER}] & [K_{EE}] \end{array}\right] \begin{Bmatrix} y_R \\ y_E \end{Bmatrix} = \begin{Bmatrix} [M_m]\{\ddot{y}_R\} \\ 0 \end{Bmatrix}$$

which is clearly not the case but from which it follows that $\{\dot{y}_E\} = - [K_{EE}]^{-1}[K_{ER}]\{\dot{y}_R\}$ in sinusoidal vibration. Substituting the above in Equation (35) gives

$$\{\dot{y}_R^T\}[M_{RR}] + \left([K_{ER}]^T [K_{EE}]^{-T} [M_{EE}] [K_{EE}]^{-1} [K_{ER}]\right) \{\dot{y}_R\}$$

$$= \{\dot{y}_R\}^T [M_m] \{\dot{y}_R\} \tag{36}$$

and the "Consistent Mass Matrix" is given by

$$[M_m] = [M_{RR}] + [K_{ER}]^T [K_{EE}]^{-T} [M_{EE}] [K_{EE}]^{-1} [K_{ER}] \tag{37}$$

This matrix is nondiagonal, like the "exact" reduced mass matrix, and has the advantage of being independent of frequency. However, comparison of Equation (37) with Equation (34) shows that the "Consistent Mass Matrix" reduces to the "exact" reduced mass matrix only at zero frequency; that is, in the static condition. As the frequency increases, the "Consistent Mass Matrix" yields increasingly erroneous results.

The reduced mass matrix in system identification is, like the others, nondiagonal and related to a model stiffness matrix which is the inverse of the RxR influence coefficients (as represented by the first R modes, which is accurate beyond direct measurement capability by many orders of magnitude); but the system identification reduced mass matrix also is independent of frequency and is exact at all the natural frequencies of the model, which are the first R natural frequencies of the helicopter. The system identification mass matrix is given by

$$[M_m] = [M_{RR}] + [\Phi_{RR}]^{-T}[\Phi_{ER}]^T[M_{EE}][\Phi_{ER}][\Phi_{RR}]^{-1} \tag{38}$$

or $[M_m] \cong [C_{RR}]^{-1}[\Phi_{RR}][\frac{1}{\Omega_R^2}][\Phi_{RR}]^{-1}$ very nearly. (39)

At the r-th natural frequency,

$$[C_{RR}]^{-1}\left[[M_{RR}] + [K_{RE}][K_{EE}]^{-1}\left(\frac{1}{\Omega_r^2}[I] - [C_{EE}][M_{EE}]\right)[K_{EE}]^{-1}[K_{ER}][C_{RR}][M_{RR}]\right]\{\phi_{Rr}\} = \{\phi_{Rr}\}\frac{1}{\Omega_r^2} \tag{40}$$

exactly. Note in Equation (38) that the reduced system identification mass matrix is expressed in terms of the modal eigenvectors of the first R modes only but includes all the masses of the actual helicopter.

Alterations in masses on the R generalized coordinates which do not affect the modal eigenvectors are, as seen from Equation (38), exactly represented. Such alterations can substantially change natural frequencies and responses. Other types of changes which do alter the modal eigenvectors may or may not be accurately reflected in the model response depending on the degree of eigenvector effects- a limit which has not been algebraically defined for any mathematical model, whether from intuitive analysis or system identification.

That such a limit should somewhere exist is a practical engineering fact. One cannot expect to obtain the equations of a sweet pea on a rubber band, then attach it to the Golden Gate bridge and expect to find the dynamic response of the bridge (the reverse, incidentally, is equally impractical). Prudence marks the boundary between utility and uselessness.

INFORMATION LOSS IN MATRIX INVERSION

It is inevitable that there will be a loss in information in numerically obtaining the response matrix from any mathematical model, or in obtaining the mathematical model from responses, even if no deliberate error is introduced.

The following is a slight modification of a derivation by Rosanoff and Ginsburg (Reference 4). Consider the equation

$$[A]\{x\} = \{b\} \tag{41}$$

in which [A] is a real symmetric nonsingular matrix. Because we calculate with numbers which have a finite number of digits, we actually solve the equation

$$\left([A] - [E]\right) \{x + \delta x\} = \{b\} \tag{42}$$

where [E] is an "error" matrix. Premultiplying both sides of Equation (42) by $[A]^{-1}$ and substituting $[A]^{-1}\{b\} = \{x\}$ gives

$$\left([I] - [A]^{-1}[E]\right) \{x + \delta x\} = \{x\} \tag{43}$$

or

$$\{\delta x\} = \left[\left([I] - [A]^{-1}[E]\right)^{-1} - [I]\right]\{x\} \tag{44}$$

Take the norm (see References 5 and 6, for example) of both sides:

$$||\{\delta x\}|| = || \left[\left([I] - [A]^{-1}[E]\right)^{-1} - [I]\right] \{x\}||$$

But the norm of the product of a matrix and a vector is less than the product of the matrix norm and the consistent vector norm:

$$||\{\delta x\}|| \leq || \left[\left([I] - [A]^{-1}[E]\right)^{-1} - [I]\right] ||\cdot||\{x\}|| \tag{45}$$

or

$$\frac{||\{\delta x\}||}{||\{x\}||} \leq || \left([I] - [A]^{-1}[E]\right)^{-1} - [I]||$$

Assume that $||[A]^{-1}[E]|| < 1$. From Faddeeva (Reference 5), it is well known that

$$|| \left([I] - [A]^{-1}[E]\right)^{-1} - \left[[I] + \left([A]^{-1}[E]\right) + \left([A]^{-1}[E]\right)^{2} \ldots + \left([A]^{-1}[E]\right)^{k}\right] || \leq \frac{||[A]^{-1}[E]||^{k+1}}{1 - ||[A]^{-1}[E]||} \quad (46)$$

if $||[A]^{-1}[E]|| < 1$. Setting $k = 0$ gives

$$|| \left([I] - [A]^{-1}[E]\right)^{-1} - [I]|| \leq \frac{||[A]^{-1}[E]||}{1 - ||[A]^{-1}[E]||} \quad (47)$$

or

$$\frac{||\{\delta x\}||}{||\{x\}||} \leq \frac{||[A]^{-1}[E]||}{1 - ||[A]^{-1}[E]||} \leq \frac{||[A]^{-1}||\cdot||[E]||}{1 - ||[A]^{-1}||\cdot||[E]||}$$

which is identical to the result obtained by Rosanoff and Ginsburg.

$$\frac{||\{\delta x\}||}{||\{x\}||} \leq \frac{||[A]||\cdot||[A]^{-1}||\cdot||[E]||/||[A]||}{1 - ||[A]||\cdot||[A]^{-1}||\cdot||[E]||/||[A]} = \frac{k_n \ell}{1 - k_n \ell} \quad (48)$$

where, following Rosanoff et al, k_n is defined as a conditioning number and ℓ as a relative error:

$$k_n \equiv ||[A]||\cdot||[A]^{-1}||$$

$$\ell \equiv ||[E]||/||[A]||$$

Taking the number of digits in the arithmetic as $\log_{10}\frac{1}{\ell}$, the reciprocal of Equation (48) gives an estimate of the number of significant digits p.

$$p = \log_{10}||x|| - \log_{10}||\delta x|| \geq \log_{10}(1 - k_n\ell) + \log_{10}\frac{1}{\ell} - \log_{10}k_n$$

but, assuming $1 \gg k_n \ell$, this estimate may be written

$$p = \log_{10} \frac{1}{\ell} - \log_{10} k_n \quad (49)$$

Thus, as shown in Reference 4, the number of information digits q lost in inverting [A] is approximately

$$q = \log_{10} k_n = \log_{10} ||[A]|| \cdot ||[A]^{-1}|| \quad (50)$$

This is true for any norm. However, the norm of a symmetrical positive definite matrix, subordinate to the Euclidian vector, is the maximum eigenvalue; and the maximum eigenvalue of the inverse is the reciprocal of the minimum eigenvalue of the matrix. Substituting this norm of [A] into Equation (50) gives the lost digits estimate.

$$q = \log_{10} \frac{\lambda(A)_{max}}{\lambda(A)_{min}} \quad (51)$$

To illustrate the immense practical importance of this, consider as an example a matrix having

$$\frac{\lambda(A)_{max}}{\lambda(A)_{min}} = 1.72 \times 10^3$$

This is the ratio of natural frequencies in the 20x20 specimen of the helicopter used in this contract. The IBM 360 uses six hexadecimal places resulting in 16^6-1 or 16777215 as the largest decimal mantissa in single precision. The inversion of the $k^{-1}m$ matrix with single precision on the computer results in an inverse having (estimated) $\log_{10}16777215 - \log_{10}1.72 \times 10^3 = 3.99$ significant digits. In other words, even starting with eight decimal places in floating point, we end up with approximately four decimal places of information in the inverse.

It is absolutely essential when dealing with test data matrices which will be inverted that the ratio of the extreme eigenvalues be minimized. Otherwise, all the physical information in the matrix is likely to be destroyed in the inversion, leaving meaningless numbers. Test data has few enough significant figures of information to begin with.

HOW TO MINIMIZE INFORMATION LOSS

A major step in this system identification process is the determination of the $\{\gamma\}$ and $\{\phi\}$ vectors by iteration. The matrix involved is the product of a mobility matrix and the inverse of another mobility matrix. This inverse presents a serious danger of information loss.

To minimize the extraneous information content of modes higher than the order of the matrix, which amounts to noise, and to narrow the spread of modal mobilities, the matrix to be inverted was made the sum of the dissipative (e.g., $[\ddot{Y}^I]$) matrices measured near each natural frequency.

Each dissipative mobility matrix has a high information content about the dominant mode and very little information about other modes. This minimizes pollution by unwanted modes but results in a very poorly conditioned matrix. For example, the 10x10 imaginary acceleration mobility of a typical helicopter measured at 3 Hz has an extreme modal mobility ratio of 10^6. However, the sum of mobility matrices over the frequency range is a matrix having the same modal vectors as a mobility matrix at any one frequency.

$$[\ddot{Y}^I_{\omega_p}] = \sum_{i=1}^{N} \ddot{Y}^{*I}_{\omega_p i} \{\phi_i\}\{\phi_i\}^T = [\Phi] \left[\ddot{Y}^{*I}_{\omega_p i} \right] [\Phi]^T \tag{52}$$

$$\sum_{\omega} [\ddot{Y}^I_{\omega}] = \sum_{i=1}^{n} \sum_{\omega} \ddot{Y}^{*I}_{\omega i} \{\phi_i\}\{\phi_i\}^T = [\Phi] \left[\sum_{\omega} \ddot{Y}^{*I}_{i} \right] [\Phi] \tag{53}$$

Therefore, Equation (53) can be used as one of the matrices in the modal eigenvector equations

$$[\sum_{\omega} \ddot{Y}^{*I}_{\omega}]^{-1} [\ddot{Y}^I_{\omega}] \{\gamma\}_i = \lambda \{\gamma\}_i \tag{54}$$

$$[\ddot{Y}^I_{\omega}] [\sum_{\omega} \ddot{Y}^{*I}_{\omega}]^{-1} \{\phi\}_i = \alpha \{\phi\}_i \tag{55}$$

The range of values from the maximum to the minimum in $\sum_{\omega} \ddot{Y}^{*I}_{\omega i}$ is very small compared to the range of $\ddot{Y}^{*I}_{\omega i}$.

If $\sum_\omega [Y^I_\omega]$ or $\sum_\omega [\dot{Y}^R_\omega]$ is used in place of $\sum_\omega [\ddot{Y}^I_\omega]$ it is necessary to normalize each of the matrices in the sum because the displacement and velocity mobilities decrease in magnitude with increased frequency. Normalization on the root mean square of the matrix elements and on the largest element absolute value were both investigated experimentally. Normalization on the RMS gave results about as satisfactory as those from acceleration mobility and is preferred over normalization on the largest element, as the latter is sensitive to errors in one term which could throw off the entire matrix. However, the differences in results, while evident, were not dramatic.

The $\log_{10}$ of the ratio of the maximum $\sum_\omega \ddot{Y}^{*I}_{\omega i}$ to the minimum was generally about .75 for the 5x5 models in these experiments and generally around 1.8 for the 15 x 15 models. The 5 x 5 models performed excellently but the 15 x 15 models performed capriciously.

If the engineer could normalize so that the matrix $[\sum_\omega \ddot{Y}^{*I}_{\omega i}]$ is unity, information would still be lost in the inversion but certainly less information than if the ratio of extreme values is very high. The $\sum_\omega \ddot{Y}^{*I}_{i}$ terms are not the eigenvalues of $\sum_\omega [\ddot{Y}^I_\omega]$. The only matrix which has a unit eigenvalue matrix is the unit matrix itself; it follows therefore that <u>some</u> information is always lost in the numerical inversion of <u>any</u> matrix other than unity.

The matrix we wish to invert is

$$\sum_\omega [\ddot{Y}^I_\omega] = [\Phi] \left[\diagdown \sum \ddot{Y}^{*I}_{\omega i} \diagdown\right] [\Phi]^T \tag{56}$$

Express the modal vector matrix in terms of its own eigenvectors [J] and its own eigenvalues λ_ϕ (that is, [J] is the eigenvector matrix of the eigenvector matrix of $[[k]^{-1}[m]]$).

$$[\Phi] = [J] [\diagdown \lambda_\phi \diagdown] [J]^{-1} \tag{57}$$

Substitute Equation (57) into Equation (56).

$$\sum_{\omega} [\ddot{Y}^I_\omega] = [J] \lceil\lambda_\phi\rfloor [J]^{-1} \lceil\Sigma \ddot{Y}^I_{\omega i}\rfloor [J]^{-T} \lceil\lambda_\phi\rfloor [J]^T \tag{58}$$

invert,

$$\sum_{\omega} [\ddot{Y}^I_\omega]^{-1} = [J]^{-T} \left\lceil \frac{1}{\lambda_\phi} \right\rfloor [J]^T \left\lceil \frac{1}{\Sigma \ddot{Y}^I_{\omega i}} \right\rfloor [J] \left\lceil \frac{1}{\lambda_\phi} \right\rfloor [J]^{-1} \tag{59}$$

The only operation on the eigenvectors [J] between Equation (58) and Equation (59) was to change relative positions; all the inversions were of diagonal matrices. As a diagonal matrix is a matrix of its own eigenvalues, having the unit matrix for eigenvectors, the central term may be treated rigorously as an eigenvalue matrix. The matrix of Equation (58) may be substituted for [A] in Equation (41), and in Equation (50), we can consider [A] as the product of the three matrices of Equation (56).

$$\sum_{\omega} [\ddot{Y}^I_\omega] = [A] = [\Phi] \lceil\Sigma \ddot{Y}^{*I}_{\omega i}\rfloor [\Phi]^T \tag{60}$$

It is well known that

$$||[A]|| \le ||[\Phi]|| \cdot ||\lceil\Sigma \ddot{Y}^{*I}_{\omega i}\rfloor [\Phi]^T|| \tag{61}$$

and that

$$||\lceil\Sigma \ddot{Y}^{*I}_{\omega i}\rfloor [\Phi]^T|| \le ||\lceil\Sigma \ddot{Y}^{*I}_{\omega i}\rfloor|| \cdot ||[\Phi]^T|| \tag{62}$$

Therefore

$$||[A]|| \le ||[\Phi]|| \cdot ||\lceil\Sigma Y^{*I}_{\omega i}\rfloor|| \cdot ||[\Phi]^T|| \tag{63}$$

At this point we wish to substitute eigenvalues, but $[\Phi]$ is not symmetric so $||[\Phi]|| \neq |\max \lambda_\phi|$. Rather, $||[\Phi]|| \geq |\max \lambda_\phi|$. Consider, therefore, the eigenvalues λ_b of $[\Phi]^T[\Phi]$ which is symmetrical.

$$[\Phi]^T[\Phi] = [L] \lceil\lambda_b\rfloor [L]^{-1} \equiv [L] \lceil\lambda_b\rfloor [L]^T \tag{64}$$

where [L] is the orthogonal matrix of eigenvectors of $[\Phi]^T[\Phi]$.

$$||[\Phi]|| = |\max\sqrt{\lambda_b}| \tag{65}$$

Substitute Equation (65) into Equation (63).

$$||[A]|| \leq |\max \lambda_b| \cdot |\max \Sigma \ddot{Y}^{*I}_{\omega i}| \tag{66}$$

Using Equation (51), the number of digits lost in inverting $\underset{\omega}{\Sigma}[\ddot{Y}_\omega]$ is approximated by

$$q \simeq \log_{10} \frac{|\max\lambda_b| \cdot |\max \Sigma \ddot{Y}^{*I}_{\omega i}|}{|\min\lambda_b| \cdot |\min \Sigma \ddot{Y}^{*I}_{\omega i}|}$$

$$= \log_{10} \frac{|\max\lambda_b|}{|\min\lambda_b|} + \log_{10} \frac{|\max \Sigma \ddot{Y}^{*I}_{\omega i}|}{|\min \Sigma \ddot{Y}^{*I}_{\omega i}|} \tag{67}$$

$\lceil\lambda_b\rfloor$ would equal a scalar times the unit matrix only if the modal vectors $\{\phi\}$ were orthogonal (i.e., $\{\phi_i^T\}\ \{\phi_j\} = 0$), a condition which could occur only in the academic cases of uniform mass: $[m] = [m][I]$. In this case, the loss of information digits would be indicated by

$$q \simeq \log_{10} \frac{|\max \ddot{Y}^{*I}_{\omega i}|}{|\min \ddot{Y}^{*I}_{\omega i}|} \tag{68}$$

and only in this case could zero information loss be achieved by normalizing the matrices such that $|\max \ddot{Y}^{*I}_{\omega i}|/|\min \ddot{Y}^{*I}_{\omega i}| = 1$. But the case is trivial, for if it were true, an inversion would be unnecessary as Φ would be the eigenvector matrix of $\underset{\omega}{\Sigma}[\ddot{Y}^*_\omega]$.

If the mass distribution is not uniform diagonal but the engineer could so normalize the matrices in the summation so that $|\max \ddot{Y}^{*I}_{\omega i}|/|\min \ddot{Y}^{*I}_{\omega i}| = 1$, it is seen from Equation (67) that there would still be a loss of information digits approximated by

$$q \simeq \log_{10} \frac{|\max \lambda_b|}{|\min \lambda_b|} \tag{69}$$

The ratio $|\max \lambda_b|/|\min \lambda_b|$ increases with the order of the mobility matrix; that is, with the number of degrees of freedom of the model. It follows, therefore, that there is an upper limit to the size of a physically meaningful reduced complete model regardless of normalization of the matrices in the summation.

As a crude "rule of thumb", Figure 1 shows the trend in the reliability of the inversion of $\sum_\omega [\ddot{Y}^I_\omega]$.

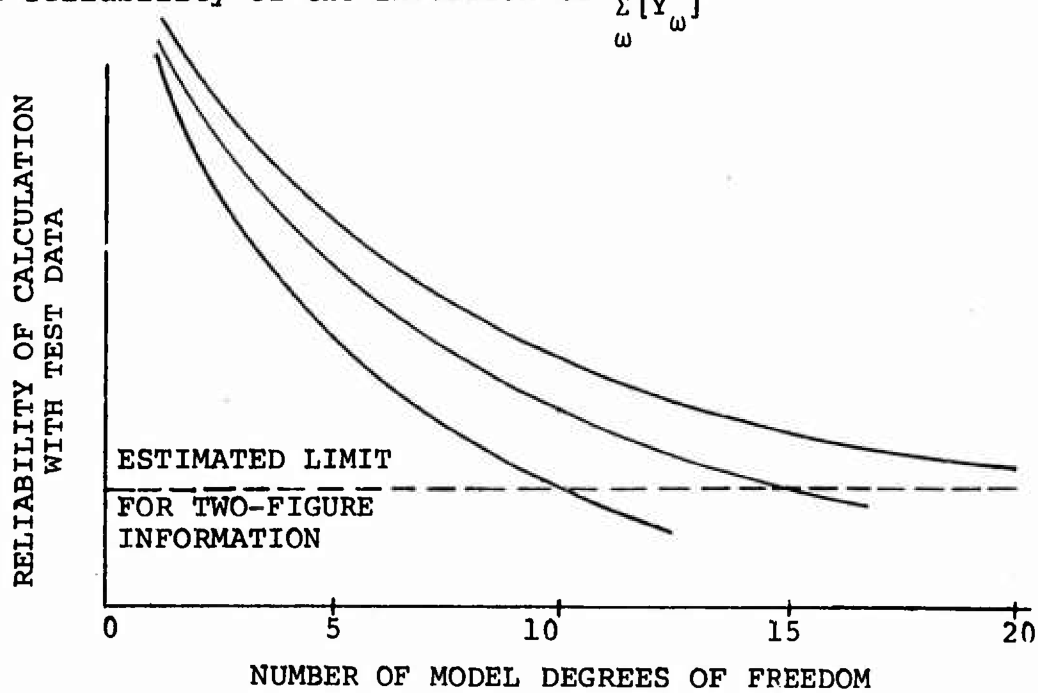

Figure 1. Reliability of the Inversion of $\sum_\omega [\ddot{Y}^I_\omega]$.

It is seen that the reliability of the calculation becomes questionable above 10 or 15 degrees of freedom. This does not mean that accurate identifications cannot be made using the iterative step for modes of, say, 20 degrees of freedom but, rather, that any one calculation has a higher probability of failure.

In passing, it should be noted that the treatment of bounds using matrix norms, as above, opens up some highly promising avenues of research on the reliability of many helicopter

theoretical calculations as well as on the reliability of the processing of test data in general. Whether, for example, some of the conventional methods of processing strain gage data yield physically meaningful results is open to question in the light of the above method.

WHEN A CALCULATION FAILS

The most common mode of failure of iteration on $[\ddot{Y}^{I}_{\omega}](\sum_{\omega}[\ddot{Y}^{I}_{\omega}])^{-1}$ is catastrophic, producing such absurd values for one or more generalized masses as negative numbers or unusually large numbers. This is signified also by a very large number of iterations required for convergence on one or more modes. Failures almost never occur with small number of degrees of freedom (e.g., five), and an identification which is quite accurate with one seed may, in the larger models, diverge with another seed.

This phenomenon results from the fact that the significant effect of error is not insidious accumulation of inaccuracies in the generalized masses but, rather, information destruction in the inversion. Fortunately, it is usually very obvious to the engineer when an identification fails on the computer, and corrective measures may be taken without rerunning the test on the helicopter.

A most obvious and effective corrective measure is to eliminate one or more of the degrees of freedom. This can be done on the computer, as the program is written so that the system may be instructed to select any of the available data which is in digital form on tape. The size of the model and the number of modes covered are consequently reduced. It is possible also to eliminate any mode, not just the highest, if it appears that a certain mode contributes little information - a local resonance, for example, in which only a small portion of the helicopter is significantly responding.

The computer experiments included a local resonance in the form of a mode in which only the most forward station showed substantial motion. When this station was not included in the identification, but the local resonance associated with its movement was included, then, as expected, there were evidences of failure in generalized mass calculation. The computer was attempting to identify a natural mode for which the input mobility data showed a largely nonresponding helicopter. This situation would be detected from the mobility plots before committing the data, as it is very apparent in the dissipative mobility spectra. The

ability to handle local resonances, or dispose of them when required, is important to a practical identification because all real structures have them. In fact, as the number of degrees of freedom of a simulated structure are realistically increased, the modal density usually increases more rapidly than the simple mathematics of uniform chains would lead one to believe. When that degree of freedom which is the predominant motion of a local resonance is eliminated, the mode it causes should be eliminated also; the mobility spectra plots for the included degrees of freedom would indicate this by an insignificant peak.

THE Γ MATRIX AND MODAL PARAMETERS

The dominant modal vector at frequency ω_i, near the i-th natural frequency Ω_i, is given by Equation (55) and the i-th gamma vector by iteration on the transpose, Equation (54). The modal mobility is obtained from

$$\{\gamma_i\}^T_{(ITR)} [\ddot{Y}^I_{\omega_i}] \{\gamma_i\}_{(ITR)} = \ddot{Y}^{*I}_{i\omega_i} \qquad (70)$$

where $\{\gamma_i\}_{(ITR)}$ is the vector from iteration (Equation 54). It is impractical to attempt the calculation using $\{\gamma_i\}$ from $[\Phi]^{-T}$ because of information loss in the inversion, as shown in Equation (59). The dominant mode is the only one used, of course, as there is negligible information content in $[\ddot{Y}^I_{\omega_i}]$ about modes other than the i-th. Therefore,

$$[\ddot{Y}^I_{\omega i}] \cong \ddot{Y}^{*I}_{i\omega_i}\{\phi_i\}\{\phi_i\}^T \approx \ddot{Y}^{*I}_{i\omega_i}\{\phi_i\}_{(ITR)}\{\phi_i\}^T_{(ITR)} \qquad (71)$$

and $\{\gamma_i\}^T_{(ITR)}\{\phi_i\}_{(ITR)} = 1$ is forced.

A peculiar situation often occurred when a calculation diverged: it was noticed that the natural frequency of the "bad" mode was usually identified with great accuracy although the calculated generalized mass was absurd, often negative, and negative calculated values of $\ddot{Y}^{*R}_{i\omega_i}$ often occurred.

The key here is the occurrence of negative values of $\dot{Y}^{*R}_{i\omega_i}$. Ideally, $[\dot{Y}^R]$ is a positive definite matrix and cannot, theoretically, be negative definite on grounds that it represents the dissipation, not a source, of energy. For any positive definite matrix B, $\{x\}^T[B]\{x\}$ is a positive number regardless of the choice of the vector $\{x\}$. The fault for negative values of $\dot{Y}^{*R}_{i\omega_i}$, which are physically impossible, cannot therefore be laid solely to $\{\gamma\}$, and therefore to the loss of information in the inverse of $\sum_\omega [\dot{Y}^I_\omega]$, because $\{\gamma_i\}^T[\dot{Y}^R]\{\gamma_i\}$ must be positive even for arbitrary $\{\gamma_i\}$ if $[\dot{Y}^R]$ is, as it is supposed to be, positive definite. We are forced to conclude that numerical errors can act in such a way as to make $[\dot{Y}^R]$ not positive definite.

The mobility $[\dot{Y}^R_{\omega_i}]$ is very nearly equal to the positive semi-definite matrix $\left[\dot{Y}^{*R}_{i\omega_i}\{\phi_i\}\{\phi_i\}^T\right]$ in which $\dot{Y}^{*R}_{i\omega_i}$ is necessarily positive. Then

$$\{\gamma_i\}^T[\dot{Y}^{*R}_{i\omega}\{\phi_i\}\{\phi_i\}^T]\{\gamma_i\} = \dot{Y}^{*R}_{i\omega}(\{\gamma_i\}^T\{\phi_i\})^2 \tag{72}$$

But $\{\gamma_i\}$ and $\{\phi_i\}$ are composed of real numbers, as opposed to imaginary or complex numbers, which makes $\{\gamma_i\}^T\{\phi_i\}$ real and $(\{\gamma_i\}^T\{\phi_i\})^2$ real and positive even for arbitrary elements in $\{\gamma\}$. The dominance of $[\dot{Y}^R_{\omega i}]$ by one mode is therefore not a cause of calculating negative values of $\dot{Y}^{*R}_{i\omega_i}$.

The calculation of absurd values of $\dot{Y}^{*R}_{i\omega}$ is nevertheless due mainly to information loss in inverting $\sum_\omega[\ddot{Y}^I_\omega]$ (or other normalized mobility matrices having similar properties), which results in poor eigenvectors in the iteration. Examination of the computer experiments shows that errors in the $[Y^R]$ or $[Y^I]$ matrices are not sufficient to cause as erratic results as have sometimes been observed if the $\{\gamma\}$ vectors in $\{\gamma\}^T[Y]\{\gamma\}$ are accurate. In the "bad" cases, the $\{\gamma\}$ vectors from iteration are invariably very bad. The reason for the occasional negative calculated values of $\dot{Y}^{*R}_{i\omega}$ is, in part,

that errors in [Y] can cause the matrix to not be positive definite. For example, in Computer Experiment 188 a nine-point identification with error yielded good results but the same identification with a different seed (Computer Experiment 184) gave poor results which included negative $\dot{Y}^{*R}_{i\omega}$ for the seventh mode. The errors by chance happened to act in such a way in Experiment 184 that excessive information was lost in the inverse, as indicated by iterations that failed to converge. The principal minor associated with the eighth and ninth positions in mobilities dominated by the seventh mode was found to be negative in the bad case (Experiment 184), due to a peculiar accumulation of random errors, which, of course, meant that the mobility was no longer positive definite, as in pure theory, and could give negative values of $\{\gamma\}^T_i[Y_{\omega_i}]\{\gamma_i\}$. However, precise $\{\gamma\}$ vectors would not have caused the negative values of $\dot{Y}^{*R}_{i\omega}$ even with [Y] not being positive semidefinite.

Calculation of physically meaningless values of $Y^{*R}_{i\omega}$, and therefore of M*, is caused primarily by information loss in inversion.

The reason for fairly accurate identifications of natural frequencies even when the generalized mass identifications are poor lies in the fact that ω_j and ω_k in Equation (22) are taken near Ω_i; therefore,

$$\frac{\omega_j \dot{Z}^{*I}_{i\omega_k} - \omega_k \dot{Z}^{*I}_{i\omega_j}}{\omega_k \dot{Z}^{*}_{i\omega_k} - \omega_j \dot{Z}^{*I}_{i\omega_j}} \simeq 1 \tag{73}$$

and

$$\Omega_i^{\ 2} = \omega_j\omega_k \frac{\omega_j \dot{Z}^{*I}_{i\omega_k} - \omega_k \dot{Z}^{*I}_{i\omega_j}}{\omega_k \dot{Z}^{*}_{i\omega_k} - \omega_j \dot{Z}^{*I}_{i\omega_j}} \simeq \omega_j\omega_k = (\Omega_i - \delta\omega_j)(\Omega_i + \delta\omega_k)$$

$$\omega_j\omega_k = \Omega_i^{\ 2} + \Omega_i(\delta\omega_k - \delta\omega_j) - \delta\omega_j\delta\omega_k$$

But $\delta\omega_k$ <<1>> $\delta\omega_j$ so $\Omega_i^{\ 2} \simeq \omega_j\omega_k$.

IDENTIFIED GENERALIZED MASSES

Typical generalized mass identifications are shown in Tables I through VI. Note in Tables I, III and V that the generalized mass of the first mode identified for a reduced model with no experimental error has always been less than the first mode generalized mass calculated from the modal vector and mass matrix of the specimen. This is not true of other modes.

Tables I and II show results of two different five-point models. No outstanding differences between the models is evident. Model 9A produced acceptable results, as shown in Table III, for different distribution of random error but Model 9B, as shown in Table IV, worked with some seeds and failed with other seeds. The failed experiments of Table IV, Computer Experiments 168 and 184, yielded drastically unrealistic values of generalized mass for most of the modes.

Table V shows a twelve-point model identification which failed only in the eighth mode. Computer Experiment 178 is identical to Computer Experiment 169 except that in the former, the computer was instructed to skip the eighth mode and, instead, operate on tape data for the thirteenth mode which resulted in satisfactory identification.

Using different stations for a twelve-point model, as shown in Table VI, produced proper identification of all models, including the eighth, with various error distributions.

Information loss in the inversion of mobility matrices is the primary cause of such failures, as shown in Computer Experiments 168, 184 and 169. The averaging of mobility test data, properly done, would greatly minimize the chances of such identification failures. Test data averaging is the customary practice. These computer experiments did not take advantage of averaging experiments.

TABLE I. IDENTIFICATION OF GENERALIZED MASSES, 5 X 5 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		152	151	157	160	182	1**
Random Amp Error		0	±5%	±5%	±5%	±5%	0
Bias Amp Error		0	+5%	+5%	+5%	+5%	0
Random Phase Error		0	±1	±1°	±1°	±1°	0
Seed		-	246	221	195	327	-
Stations (In.)	Mode	Generalized Masses ($Lb\text{-}Sec^2/In.$)					
0	1	7.9910	7.3594	7.3834	7.8421	8.2330	8.5341
120	2	4.6248	3.8247	4.5951	4.1440	4.0594	4.4491
220	3	.4951	.4618	.4771	.4653	.4729	.4951
340	4	1.0897	1.0372	1.0657	1.0366	1.0440	1.0872
460	5	.6463	.5869	.6247	.6691	.6131	.6302

* Model 5A

** From 20 x 20 Model

TABLE II. IDENTIFICATION OF GENERALIZED MASSES, 5 X 5 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		159	170	183	1**
Random Amp Error		±5%	±5%	±5%	0
Bias Amp Error		+5%	+5%	+5%	0
Random Phase Error		±1°	±1°	±1°	-
Seed		221	246	128	-
Stations (In.)	Mode	Generalized Masses ($Lb\text{-}Sec^2/In.$)			
0	1	7.4385	7.3210	7.8179	8.5341
100	2	4.4545	4.1824	4.3797	4.4491
200	3	.4724	.4620	.4596	.4951
320	4	1.0769	1.0277	1.0233	1.0872
460	5	.6912	.5945	.6360	.6302

* Model 5B

** From 20 x 20 Model

TABLE III. IDENTIFICATION OF GENERALIZED MASSES, 9 X 9 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		180	156	162	179	187	1**
Random Amp Error		0	±5%	±5%	±5%	±5%	0
Bias Amp Error		0	+5%	+5%	+5%	+5%	0
Random Phase Error		0	±1°	±1°	±1°	±1°	0
Seed		-	287	50	315	492	-
Stations (In.)	Mode	Generalized Masses ($Lb\text{-}Sec^2/In.$)					
0	1	7.9538	7.3776	7.5946	8.2378	7.4531	8.5342
30	2	4.5889	4.1130	4.2450	4.6233	4.2020	4.4491
100	3	.4938	.4671	.4821	.4614	.4656	.4951
160	4	1.0863	1.0507	1.0368	1.0129	1.0785	1.0872
220	5	.6350	.6164	.6044	.6102	.5971	.6302
280	6	.7457	.7049	.6983	.7227	.7239	.7429
340	7	1.1746	1.1204	1.1332	1.1064	1.0968	1.1769
400	8	1.5002	1.3770	1.4070	1.4193	1.4783	1.4683
460	9	.6593	.6576	.5507	.6235	.5737	.7866

* Model 9A

** From 20 x 20 Model

TABLE IV. IDENTIFICATION OF GENERALIZED MASSES, 9 X 9 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		161	188	168	184	1**
Random Amp Error		±5%	±5%	±5%	±5%	0
Bias Amp Error		+5%	+5%	+5%	+5%	0
Random Phase Error		±1	±1	±1	±1	-
Seed		287	206	395	619	-
Stations (In.)	**Mode**	**Generalized Masses ($Lb\text{-}Sec^2/In.$)**				
0	1	7.4445	7.5891	7.6797	7.0969	8.5341
60	2	4.2851	4.4084	23.2234	4.5730	4.5615
120	3	.4741	.4545	.6876	.4314	.4951
180	4	1.0194	1.0226	28.5896	1.0968	1.0872
240	5	.6343	.6740	.5667	-7.9847	.6302
280	6	.7020	.6987	-8.5143	.5237	.7429
320	7	1.1877	1.0711	-.0080	.0125	1.1769
400	8	1.2510	1.7815	.1256	-.2199	1.4683
460	9	.9347	.9398	-.0159	-.0810	.9836

* Model 9B

** From 20 x 20 Model

TABLE V. IDENTIFICATION OF GENERALIZED MASSES, 12 X 12 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		169	178	1**
Random Amp Error		±5%	±5%	0
Bias Amp Error		+5%	+5%	0
Random Phase Error		±1°	±1°	0
Seed		492	492	-
Stations (In.)	Mode	Generalized Masses ($Lb\text{-}Sec^2/In.$)		
0	1	7.4551	7.4629	8.5341
60	2	4.1298	3.9789	4.4491
100	3	.4587	.4657	.4951
120	4	1.0446	1.0376	1.0872
160	5	.5950	.5802	.6302
200	6	.6869	.6975	.7429
240	7	1.2036	1.2044	1.2569
280	8	-7.9616		2.0521
320	9	.9410	.9118	.9836
370	10	.0425	.0428	.0432
430	11	.1718	.1752	.1723
460	12	1.0012	1.0037	1.0480
	13	-	.7924	.5724

* Model 12D

** From 20 x 20 Model

TABLE VI. IDENTIFICATION OF GENERALIZED MASSES, 12 X 12 MODEL* OF 20 X 20 SPECIMEN

Computer Experiment Number		150	149	155	163	1**
Random Amp Error		0	±5%	±5%	±5%	0
Bias Amp Error		0	+5%	+5%	+5%	0
Random Phase Error		0	±1°	±1°	±1°	0
Seed		-	23	492	87	-
Stations (In.)	Mode	Generalized Masses ($\text{Lb-Sec}^2/\text{In.}$)				
0	1	7.9718	7.7160	7.2917	7.4071	8.5342
30	2	4.6071	4.5010	4.2722	4.3406	4.4491
60	3	.4941	.4640	.4682	.4611	.4951
100	4	1.0857	1.0499	1.0625	1.0425	1.0872
140	5	.6348	.6094	.5958	.5936	.6302
180	6	.7441	.7155	.6930	.7097	.7429
220	7	1.1765	1.1433	1.1101	1.1278	1.1769
260	8	1.4158	1.3467	1.3225	1.3454	1.4115
300	9	.7808	.7329	.7395	.7362	.7866
340	10	.0430	.0419	.0422	.0422	.0432
400	11	.1705	.1596	.1665	.1689	.1723
460	12	.9112	.5712	.8417	1.0946	1.3235

* Model 12B

** From 20 x 20 Model

RESPONSE FROM IDENTIFIED MODEL

Figures 2 through 7 portray typical acceleration response obtained from the various models investigated in the present study. In each instance, the exact curve was obtained from the twenty-point structure with zero error. Figure 2 indicates the effect of random number seed for a typical five-point model. Figure 3 presents the results obtained for one of the nine-point models considered in the investigation. Figure 4 portrays the effect of random number seed on the twelve-point model. All the computer experiments which considered error used a $\pm 5\%$ random, 5% bias and a 1° phase error.

Figure 5 presents the effect of model variation on the acceleration response. The models varied in that different spanwise masses were considered. Model 5A utilized stations 0, 120, 220, 340 and 460 (inches) whereas model 5B consisted of stations 0, 100, 200, 320, and 460 (inches). Figure 6 presents the effect of model for the nine-point model. The model 9A consisted of stations 0, 30, 100, 160, 220, 280, 340, 400 and 460 (inches). Model 9B included stations 0, 60, 120, 180, 240, 280, 320, 400 and 460 (inches). The twelve-point model 12B used stations 0, 30, 60, 100, 140, 180, 220, 260, 300, 340, 400 and 460 (inches) whereas model 12E utilized stations 0, 30, 60, 100, 120, 160, 200, 260, 280, 340, 400, 460 (inches). For each model the computer experiments were executed using the same random number seed and the aforementioned errors were incorporated.

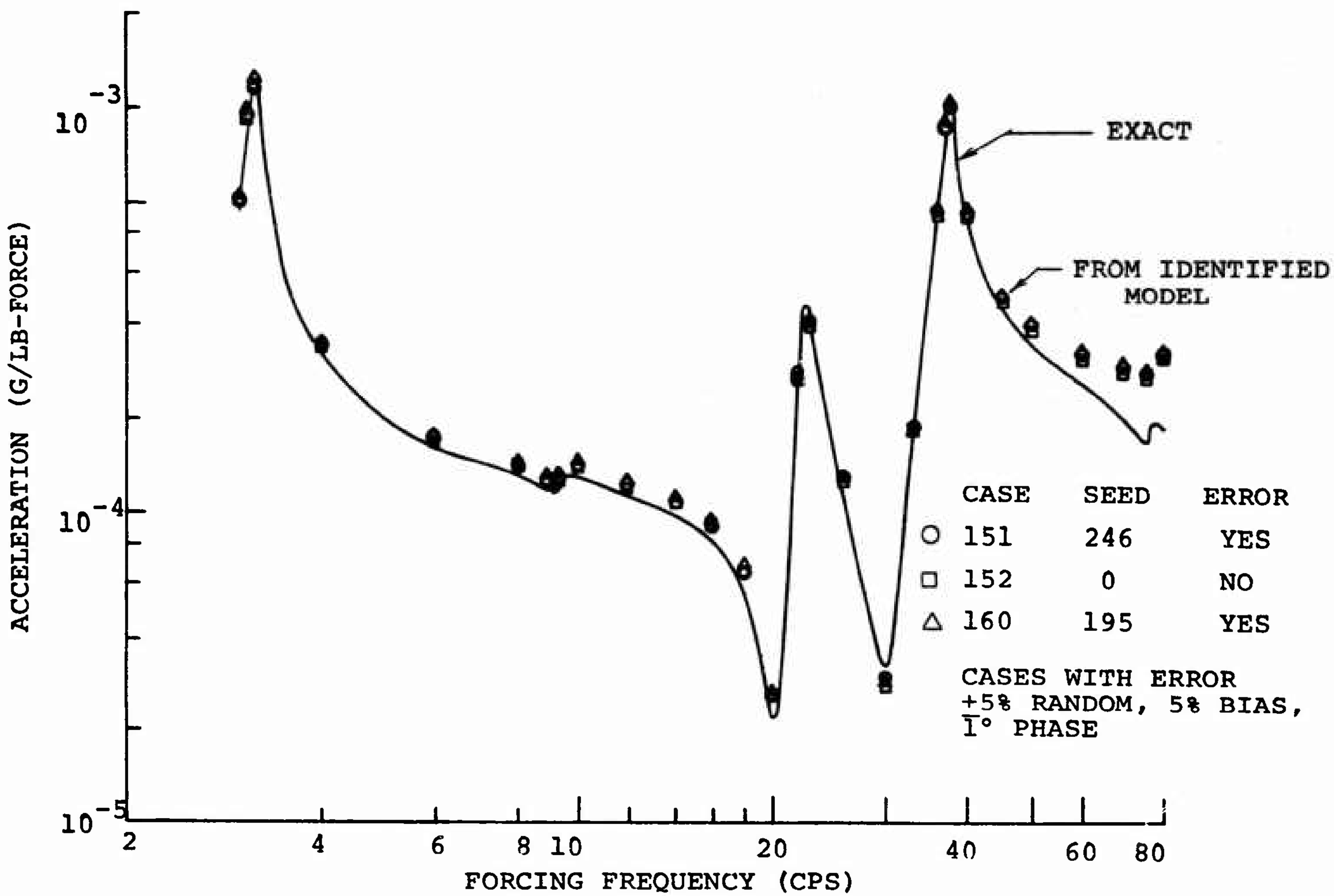

Figure 2. Five-Point Model Response Obtained From Equations With Identified Parameters; Driving Point at Hub.

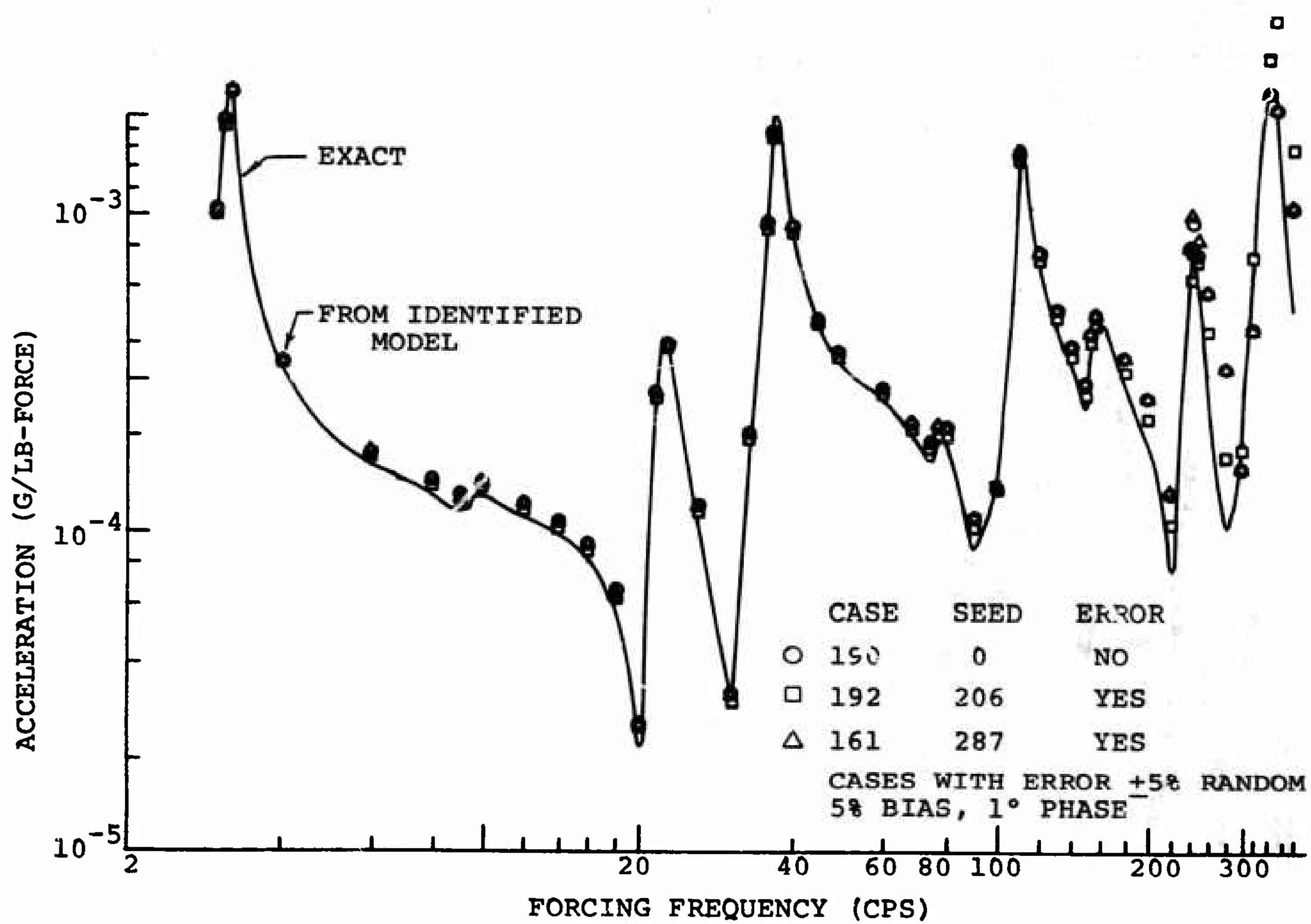

Figure 3. Nine-Point Model Response Obtained From Equations With Identified Parameters; Driving Point at Hub.

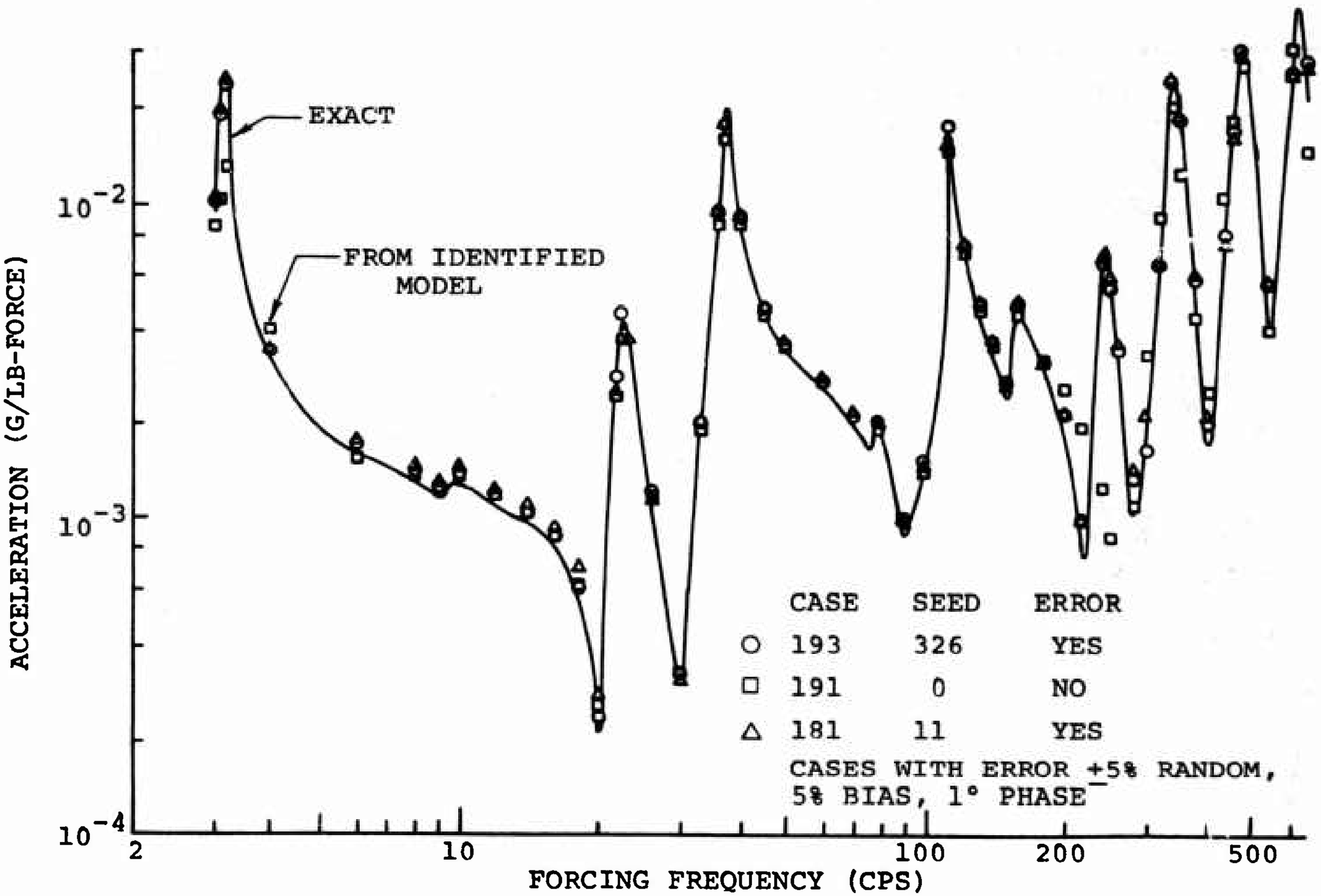

Figure 4. Twelve-Point Model Response Obtained From Equations With Identified Parameters; Driving Point at Hub.

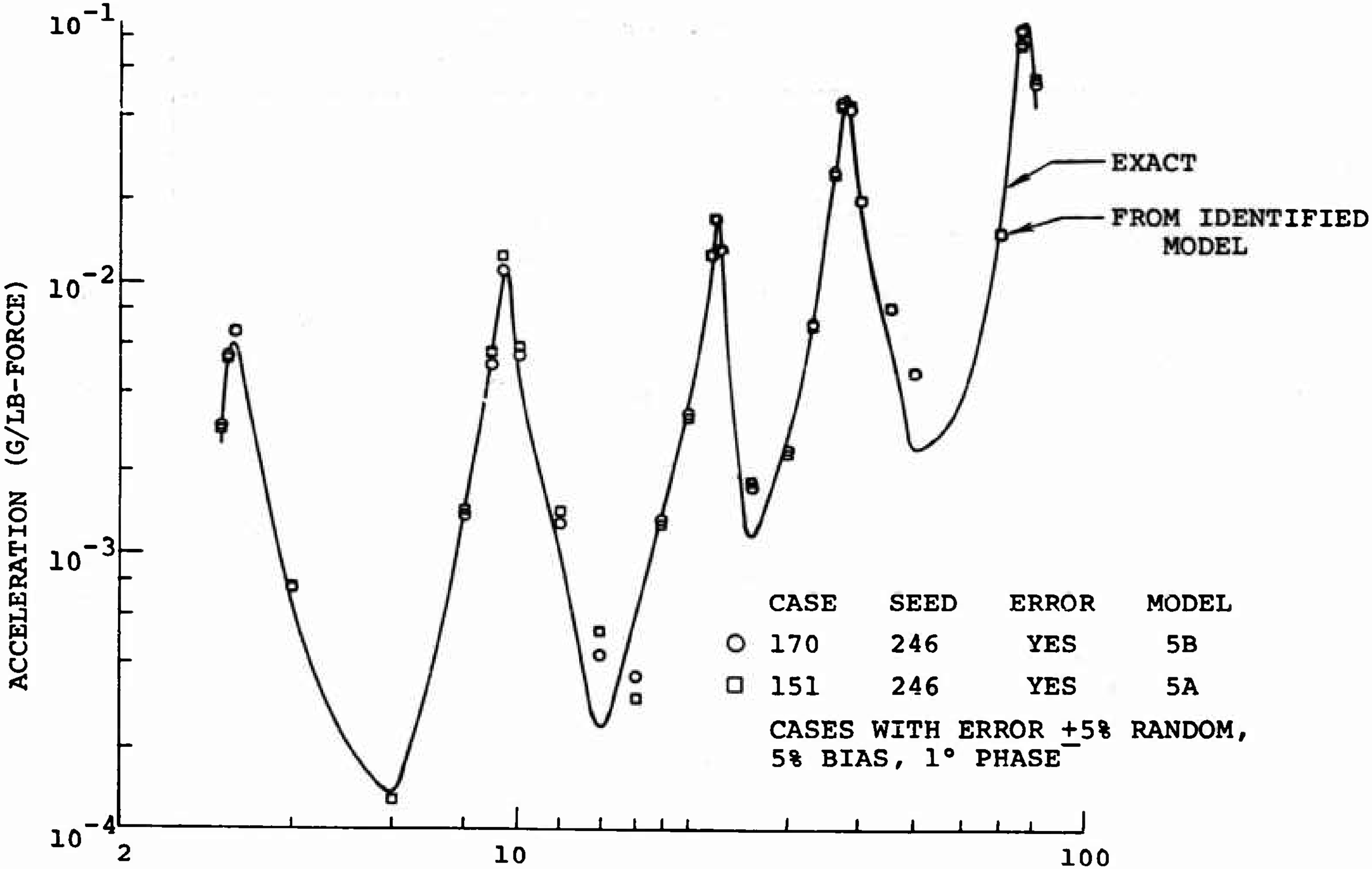

Figure 5. Five-Point Model Response, Effect of Model; Driving Point at Station 1.

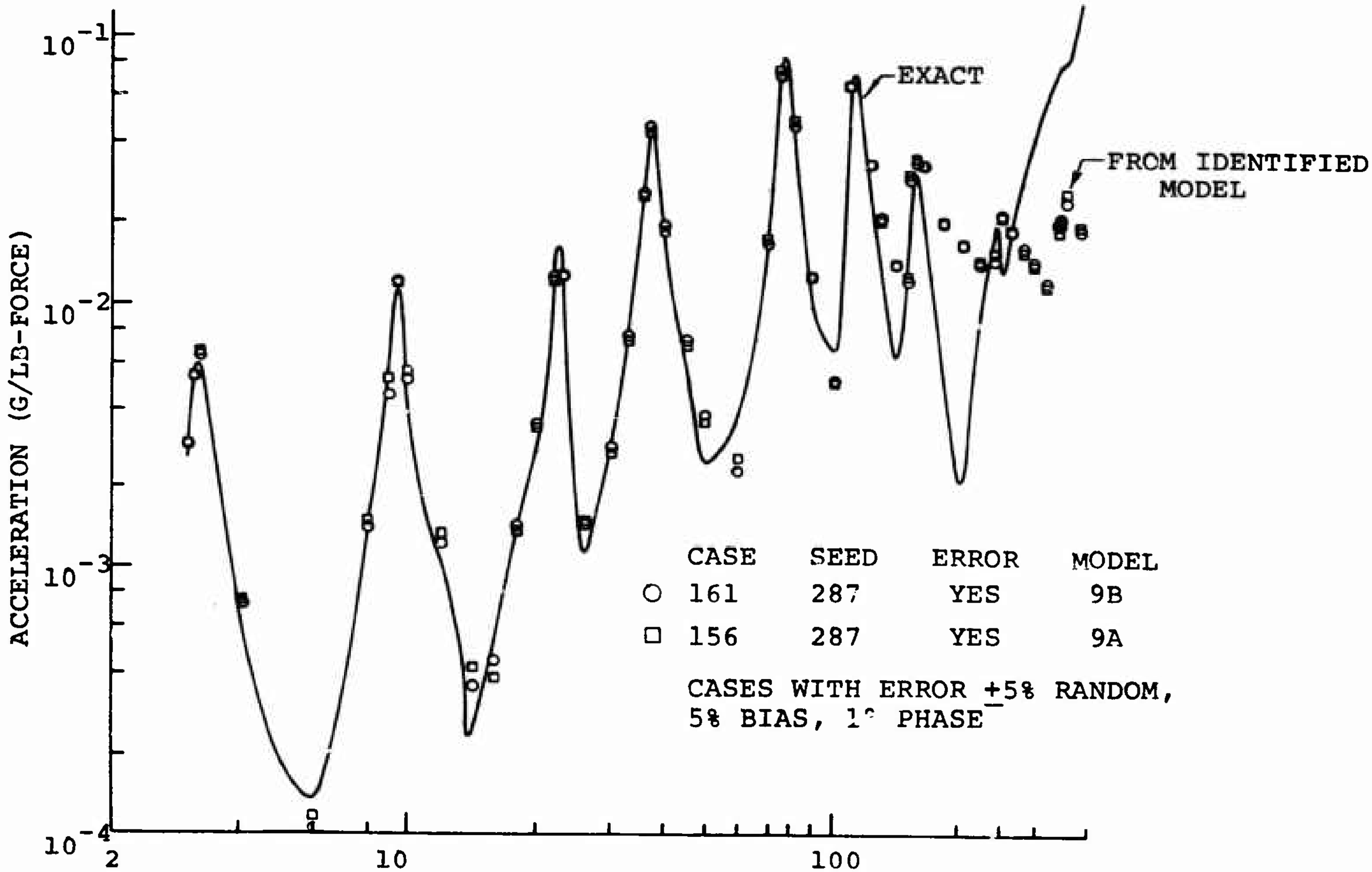

Figure 6. Nine-Point Model Response, Effect of Model; Driving Point at Station 1.

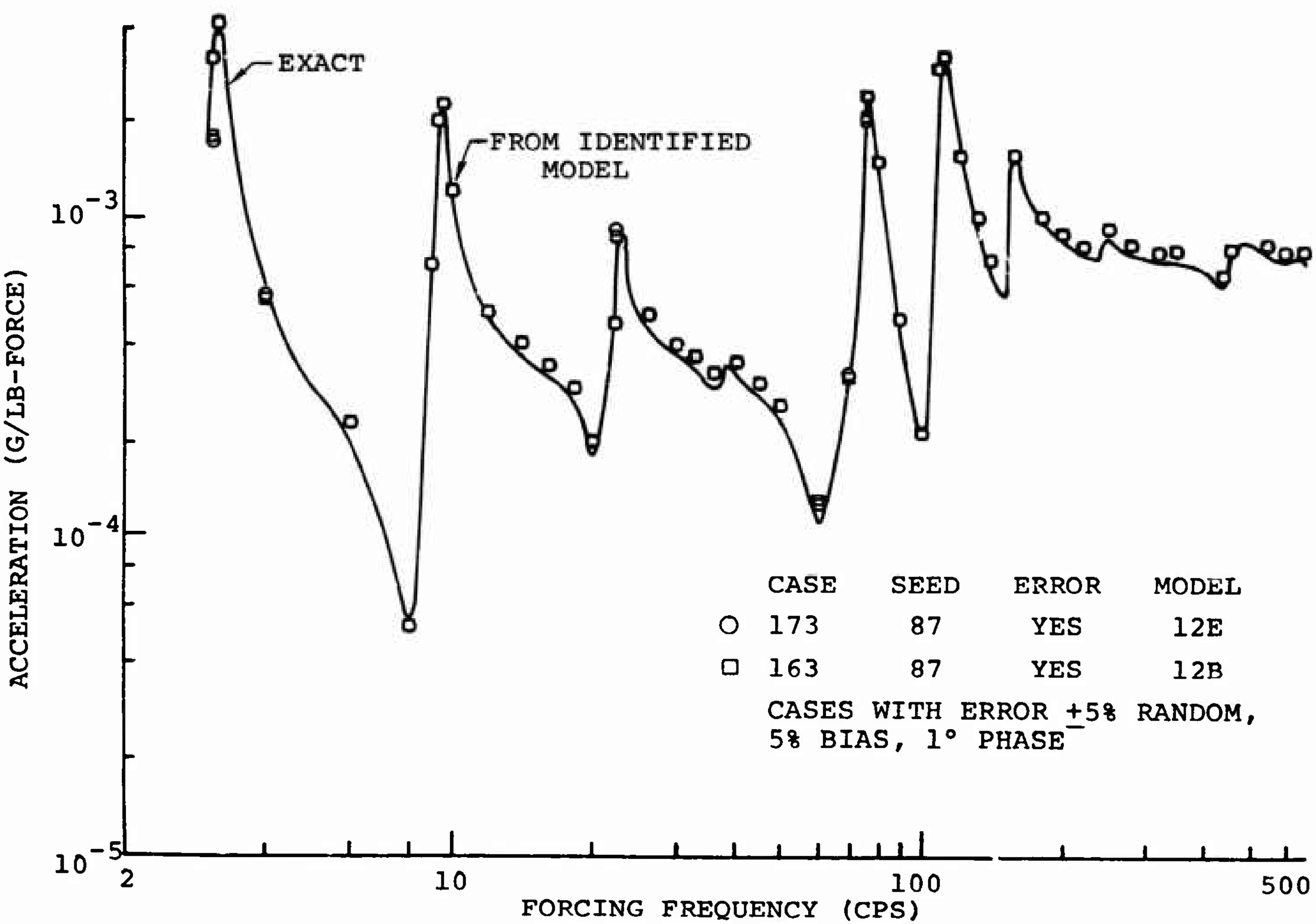

Figure 7. Twelve-Point Model Response, Effect of Model; Driving Point at Station 3.

CONCLUSIONS

1. The equations of motion for a structure may be obtained using only impedance-type test data without the use of an intuitive mathematical model.

2. The method also yields the eigenvector or mode shape and generalized mass corresponding to each natural frequency.

3. The accuracy of the dynamic response of a structure using impedance-type experimental data is not dependent on the accuracy of the test measurements, provided the data is within the state of the measurement art.

4. The mass matrix assumed for an intuitive mathematical model should be fully populated to yield accurate dynamic response results.

5. To insure minimum information loss in the inversion of mobility matrices, the averaging of mobility test data should be used in practice.

6. There is an upper limit to the size of a physically meaningful reduced complete model yielding minimum loss of information digits. The present report indicates the maximum to be a model of approximately 15 degrees of freedom.

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APPENDIX
COMPUTER PROGRAM DESCRIPTION

Note: All integer variables must be right justified with no decimal point.

Tape, Card Reader and Printer Assignments.

1 Card Reader

3 Printer

9 Contains influence coefficient matrix for use in XACT.

10 Tape assignment in XACT program. Contains mobility data for all degrees of freedom, with no error for specified frequencies for use in INXACT program.

11 Tape assignment in INXACT program. Contains mobility data with reduced stations and error (i.e., simulated test data) for use in program IDENTRE.

All input data must be in the following units.

Mass - lb-sec^2/in.
Stiffness - lb/in.
Frequencies - Hz

PROGRAM XACT

Card	1	Columns	1	IC	Program Control IC $\neq$ 0 End Program IC = 0 Continue Program
			2-80	HEAD	Case Description
Card	2	Columns	1-10	ND	Number of Degrees of Freedom ($\leq$ 20)
			11-20	G	Structural Damping Coefficient
			21-30	NC	Number of Modes to be Obtained From Matrix Product [C][M]. If NC = 0, K is not inverted.
			31-40	NK	Number of Modes to be Obtained from Matrix Product $[K]^{-1}[M]$.
Card(s)	3			M	Mass Matrix. (8E10.0 Format). For full symmetric matrix load lower triangular matrix only starting each row on a new card and ending with the diagonal element. Use as many cards as necessary. For a diagonal mass matrix, load one blank card followed by cards containing diagonal elements in sequence (8E10.0 Format).
Card(s)	4			K	For direct loading of K matrix from cards, proceed as for M matrix as described above.
C Matrix Option				C	To load C matrix from TAPE 9, load one blank card. This will read C matrix from TAPE 9. Unformatted record contains heading (20 words, first character blank); NX (order of matrix). Force deflection influence coefficient matrix.

Card	5	Columns	1-5	NF	Number of Frequencies Used ($\leq$ 100)
			6-10	IP1	Print Control of Data Written on TAPE 10. IP1 = 0 No Printed Output Except List of Frequencies IP1 = 1 Print Full Mobility Matrix, Real and Imaginary at Each Frequency IP1 = 2 Print Only Diagonal Elements and Row of Mobility Matrix, Real and Imaginary at Each Frequency
		Columns	11-15	IP2	Control on Printed Output IP2 = 0 Same as Written on Tape Above, Complex Velocity Mobility Matrix at Each Frequency IP2 = 1 Print Acceleration Amplitude and Phase Angle
			16-20	NRØW	This is the row to be printed when IP2 = 2. If NRØW = 0 then only diagonal (driving point) elements are printed as output.
Card(s) Omit if NF = 0	6			HZ	Frequencies in Hertz. 10 Columns Per Value, 8 Values Per Card (100 Maximum). Format (8F10.2)
Card	7				Frequency sweep control. This card is the same as Card 5 except that TAPE 10 is not written. To get response data with no tape use a blank card for Card 5 followed by Card 6. To generate TAPE 10 and print no other response data follow Card 5 by one blank card for Card 6. Both options indicated by Card 5 and Card 6 may be used simultaneously.
Card	8	Column			For termination of Case Use 1 in Column 1. Blank card indicates another case to follow, beginning with card 1 again.

PROGRAM INXACT

Card	1	Column	1	IC	Program Control
			2-80	HEADN	Case Description
Card	2	Columns	1-10	NR	Number of Points Tested (Number of Degrees of Freedom of the Model)
			11-20	PCT	Random Error Applied to Amplitude, Uniform between - and + PCT* Element Amplitude.
			21-30	PCTB	Bias Error Applied to Amplitude. PCTB* Element Amplitude.
			31-40	PHE	Random Error in Degrees Applied to Phase Angle. Uniform Between -PHE and +PHE.
			71-80	IZ	Random Number Seed.
Card	3			KEEP	Stations to be used in model. Card 3 is included only if NR < ND (From Program XACT). Five columns per value, maximum of 10 values per card (Format 10I5)
Card	4	Columns	1-5	NFR	Number of Frequencies to be Used (From TAPE 10, XACT Program) IF NFR = 0 all frequencies on TAPE 10 are to be used.
			6-10	IP1	Same Definitions
			11-15	IP2	as in
			16-20	NRØW	XACT Program
Card(s)	5			INDX	Indices of Frequencies to be Used from TAPE 10 XACT Program. Indices must be in ascending order. Five columns per value, 16 values per card (Format 16I5).

PROGRAM IDENTRE

Card	1	Columns	1	IC	Program Control IC = 1 Full Program Output IC > 1 Terminate Program
			2-80	HEADI	Case Description
Card	2	Column	1	NNØR	Control on Normalization of Mobility Matrices
Card(s)	3			INDX	Indices of the Frequencies on TAPE 11 From INXACT Program to be Used in Summation of Real Parts of Mobility Elements (NFR Frequencies. Must be in Ascending Order) Five Columns Per Value, 16 Values Per Card (Format 16I5)
Card(s)	4			INDX	Indices of Frequencies to be Used for PHI Iteration (MODE SHAPE). Same Number of Indices as the Number of Degrees of Freedom of the Model. Indices in Ascending Order.
Card(s)	5			IØM	Indices of Frequencies to be Used in Forming $Y^*_{i(\omega)}$ and $Z^*_{i(\omega)}$ in the Calculation of Generalized Mass and Natural Frequency (2* Number of Degrees of Freedom of the Model). Indices in Ascending Order.

Card	6	Columns	1-5	NF	No. of Frequencies at Which Reidentification of Mobility Matrices is Calculated.
			6-10	IP1	Print Control of Mobility Data IP1 = 0 No printed output except list of frequencies IP1 = 1 Full matrices printed IP1 = 2 Diagonal elements and row printed
			11-15	IP2	IP2 = 0 Complex velocity mobilities printed IP2 = 1 Acceleration mobilities printed Amplitude in g's and phase in degrees
				NROW	This is Row to be Printed when IP1 = 2. If Equal to Zero the Only Diagonal (Driving Point) Elements are Printed
			16-20	NN	Controls Type of Damping Used in Reidentification of Mobilities NN = 0 Use Scalar Structural Damping Coefficient *K Matrix NN = 1 Use Damping Matrix
Card(s)	7			HZ	Frequencies at Which Reidentification is Calculated Ten Columns Per Value, 8 Values Per Card (Format 8F10.0).
Card	8	Column	1		A 2 in Column 1 Terminates Program Otherwise Return to Card 1 for Beginning of New Case

COMPUTER PROGRAM FORTRAN LISTING

```
C        XACT  XACT  XACT  XACT  XACT  XACT  XACT  XACT  XACT  XACT
      INTEGER HEAD(20),HEAD1(20),IT(20),ITK(20)
       INTEGER HT(7)
      REAL M(20,21),K(20,21),C(20,21),A(20,21),B(20,21),PHI(2.,21),
     A FRE(20),DUM(20),GM(20),MU(20,21),PHIK(20,21),FREK(20),GNK(20)
      REAL MZ(100),ZR(20,21),ZI(20,21),YR(20,21),YI(20,21),DPR(100,20),
     A DPI(100,20),TR(100,20),TI(100,20)
      LOGICAL TORF,TAPE
      DATA HT/'EXAC','T DA','TA S','IMUL','ATED',' TES','T   '/
     0
C                       READ INPUT
  100 READ (1,110) IC,HEAD
  110 FORMAT (I1,A3,19A4)
      IF (IC.NE.0) GO TO 700
      READ (1,120) ND,G,NC,NK
  120 FORMAT (I10,F10.0 ,2I10)
      ND1=ND-1
      READ (1,130) M(1,1)
  130 FORMAT (8E10.0)
      IF(M(1,1).NE.0) GO TO 150
C                       DIAGONAL MASS
      DO 140 I=1,ND
      DO 140 J=1,ND
  140 M(I,J)=0
      READ (1,130) (M(I,I),I=1,ND)
      GO TO 170
C                       FULL MASS MATRIX
  150 DO 160 I=2,ND
  160 READ (1,130) (M(I,J),J=1,I)
      CALL SYM (M,ND)
  170 READ (1,130) K(1,1)
      IF (K(1,1).EQ.0) GO TO 190
C                       K INPUT
      DO 180 I=2,ND
  180 READ (1,130) (K(I,J),J=1,I)
      CALL SYM (K,ND)
      GO TO 230
C                       C FROM TAPE
  190 READ (9) HEAD1,NX,((C(I,J),I=1,NX),J=1,NX)
      IF (ND.EQ.NX) GO TO 210
      WRITE (3,200) HEAD,HEAD1
  200 FORMAT (10X,A3,19A4//T5,'C MATRIX WRONG SIZE'//T5,'TAPE HEADING',
     A 10X,1H'A3,19A4,1H')
      CALL EXIT
C                       INVERT AND SYMMETRIZE C
  210 CALL INVRS (C,ND,K)
      DO 220 I=1,ND1
      I1=I+1
      DO 220 J=I1,ND
      K(I,J)=(K(I,J)+K(J,I))/2.0
  220 K(J,I)=K(I,J)
C                       INVERT K
  230 IF (NX.EQ.0.AND.NC.NE.0) CALL INVRS (K,ND,C)
C                       SUM K ROWS
  240 DO 250 I=1,ND
```

```
      K(I,21)=0
      DO 250 J=1,ND
  250 K(I,21)=K(I,21)+K(I,J)
C                                LIST INPUT
      WRITE (3,260) HEAD,ND,G
  260 FORMAT ('1'/T5 ,20(' XACT ')//T5,15('*'),5X,A3,19A4,5X,15('*')//
     A I10,'  DEGREES OF FREEDOM'10X,'STRUCTJRAL DAMPING PARAMETER = '
     B F6.3//T50,'MASS MATRIX'/)
      CALL MOUT2 (M,ND,ND)
      IF (NX.EQ.0.AND.NC.EQ.0) GO TO 290
      WRITE (3,270)
  270 FORMAT ('1'//T45,'INFLUENCE COEFFICIENT MATRIX'//)
      IF (NX.NE.0) WRITE (3,280)  HEAD1
  280 FORMAT ('+'T75,'FROM TAPE'//T5,'TAPE HEADING'10X,1H'A3,19A4,1H'//)
      CALL MOUT2 (C,ND,ND)
  290 WRITE (3,300)
  300 FORMAT ('1'//T50,'STIFFNESS MATRIX'//)
      CALL MOUT2 (K,ND,ND)
      WRITE (3,310) (K(I,21),I=1,ND)
  310 FORMAT (//T50,'SPRINGS TO GROUND'//(T10,1P10E12.4))
      IF (NC.EQ.0) GO TO 350
C                                CALC FREQ AND MODES     C*M    FRE IN HZ
      DO 320 I=1,ND
      DO 320 J=1,ND
  320 B(I,J)=C(I,J)
      DO 340 J=1,NC
      CALL MMPY (B,M,ND,ND,ND,A)
      CALL SITER (A,PHI,FRE,J,ND,ITN,PMAX)
      FRE(J)=FRE(J)/6.283185
      IT(J)=ITN
      DO 330 I=1,ND
  330 DUM(I)=PHI(I,J)
      GM(J)=GEN(DUM,M,ND)
      CON=PMAX/GM(J)
      DO 340 I=1,ND
      DO 340 L=1,ND
  340 B(I,L)=B(I,L)-DUM(I)*DUM(L)*CON
  350 IF(NK.EQ.0) GO TO 390
C                                FREQ AND MODES      MU * K     FREK IN HZ
      DO 360 I=1,ND
      DO 360 J=1,ND
  360 B(I,J)=K(I,J)
      CALL INVRS (M,ND,MU)
      CALL MMPY (MU,B,ND,ND,ND,A)
      CALL MMPY (A,MU,ND,ND,ND,B)
      DO 380 J=1,NK
      CALL MMPY (B,M ,ND,ND,ND,A)
      CALL SITER (A,PHIK,FREK,J,ND,ITN,PMAX)
      FREK(J)=1./FREK(J)/6.283185
      ITK(J)=ITN
      DO 370 I=1,ND
  370 DUM(I)=PHIK(I,J)
      GMK(J)=GEN(DUM,M,ND)
      CON =PMAX/GMK(J)
      DO 380 I=1,ND
```

```
      DO 380 L=1,ND
  380 B(I,L)=B(I,L)-DUM(I)*DUM(L)*CON
  390 IF (NC.EQ.0) GO TO 430
C                               MODAL OUTPUT
      WRITE (3,400)
  400 FORMAT ('1'//T45,'NORMAL MODES FROM C MATRIX'//)
      CALL MOUT2 (PHI,ND,NC)
      WRITE (3,410) (FRE(I),I=1,NC)
  410 FORMAT (//T45,'FREQUENCIES - HZ'//(T10,10F12.6))
      WRITE (3,420) (GM(I),I=1,NC)
  420 FORMAT (//T45,'GENERALIZED MASS'//(T10,10F12.6))
  430 IF (NK.EQ.0) GO TO 450
      WRITE (3,440)
  440 FORMAT ('1'//T45,'NORMAL MODES FROM K MATRIX'//)
      CALL MOUT2 (PHIK,ND,NK)
      WRITE (3,410) (FREK(I),I=1,NK)
      WRITE (3,420) (GMK(I),I=1,NK)
C                               READ TAPE CONTROLS
  450 TAPE=.TRUE.
  460 READ (1,470) NF,IP1,IP2,NROW
  470 FORMAT (4I5)
      IF (.NOT.TAPE.AND.IP1.EQ.0) GO TO 100
      IF (NF.EQ.0) GO TO 690
      TORF=NROW.GT.0.AND.NROW.LE.ND
      READ (1,130) (HZ(I),I=1,NF)
C                               FORM MOBILITY AND WRITE TAPE
      IF (TAPE) WRITE (10) HT,HEAD,NF,ND,(HZ(I),I=1,NF)
      DO 570 L=1,NF
      CALL MOB (M,K,G,ND,HZ(L),ZR,ZI,YR,YI)
       IF(TAPE) WRITE (10) HZ(L),((YR(I,J),YI(I,J),I=1,ND),J=1,ND)
      IF(IP1-1) 570,480,550
  480 IF(IP2.NE.0) CALL MATAMP (HZ(L),YR,YI,ND)
      IF (TAPE) WRITE (3,490)
  490 FORMAT ('1'T10,'COMPLEX MOBILITY WRITTEN ON TAPE'/)
      IF (.NOT.TAPE) WRITE (3,540)
      IF(IP2.NE.0) GO TO 510
      WRITE (3,500) HZ(L)
  500 FORMAT (/T40,'REAL MOBILITY, IMAGINARY MOBILITY      FREQ ='F10.2,
     A ' HERTZ'//)
      GO TO 530
  510 WRITE (3,520)  HZ(L)
  520 FORMAT (/T40,'ACCELERATION AMPLITUDE IN G''S, PHASE IN DEG.   FREQ
     A ='F10.2,' HERTZ'//)
  530 CALL MOUT2 (YR,ND,ND)
      WRITE (3,540)
  540 FORMAT ('1'//)
      CALL MOUT2 (YI,ND,ND)
      GO TO 570
  550 DO 560 I=1,ND
      DPR(L,I)=YR(I,I)
      DPI(L,I)=YI(I,I)
      IF(.NOT.TORF) GO TO 560
      TR(L,I)=YR(NROW,I)
  560 TI(L,I)=YI(NROW,I)
  570 CONTINUE
```

```
      IF (IP1-1) 580,690,600
  580 WRITE (3,590) (HZ(I),I=1,NF)
  590 FORMAT (///T10,'MOBILITY MATRICES AT THE FOLLOWING FREQUENCIES (HZ
     A) HAVE BEEN WRITTEN ON TAPE'//(T10,10F12.6))
      GO TO 690
  600 IF(IP2.NE.1) GO TO 620
      CALL AMP (HZ,DPR,DPI,NF,ND)
      IF(TORF) CALL AMP (HZ,TR,TI,NF,ND)
      WRITE (3,610)
  610 FORMAT ('1'T40,'DRIVING POINT RESPONSE,  AMP IN G''S AND PHASE IN
     ADEGREES'//)
      GO TO 640
  620 WRITF (3,630)
  630 FORMAT ('1'T40,'DRIVING POINT MOBILITY,  REAL AND IMAGINARY'//)
  640 CALL YOUT (HZ,DPR,NF,ND,0)
      WRITE (3,540)
      CALL YOUT (HZ,DPI,NF,ND,IP2)
      IF(.NOT.TORF) GO TO 690
      IF (IP2.NE.1) GO TO 660
      WRITE (3,650) NROW
  650 FORMAT ('1'T30,'TRANSFER RESPONSE, ROW 'I5,'  AMP IN G''S AND PHAS
     AE IN DEG'//)
      GO TO 680
  660 WRITE (3,670) NROW
  670 FORMAT ('1'T30,'TRANSFER MOBILITY, ROW 'I5,' REAL AND IMAG'//)
  680 CALL YOUT (HZ,TR,NF,ND,0)
      WRITE (3,540)
      CALL YOUT (HZ,TI,NF,ND, IP2)
  690 IF (.NOT.TAPE) GO TO 100
      TAPE = .FALSE.
      GO TO 460
  700 REWIND 10
      CALL EXIT
      END
```

```
      SUBROUTINE SYM (A,N)                                              SYM   1
C               FORMS SYMMETRIC MATRIX FROM LOWER TRIANGLE              SYM   2
      REAL A(20,21)                                                     SYM   3
      N1=N-1                                                            SYM   4
      DO 100 I=1,N1                                                    1SYM   5
      I1=I+1                                                           1SYM   6
      DO 100 J=I1,N                                                    2SYM   7
  100 A(I,J)=A(J,I)                                                    2SYM   8
      RETURN                                                            SYM   9
      END                                                               SYM  10
```

```
      SUBROUTINE MOUT2 (A,M,N)                                          MOT   1
      REAL A(20,21)                                                     MOT   2
      ID=MINO(N,10)                                                     MOT   3
      WRITE (3,100) (I,I=1,ID)                                          MOT   4
  100 FORMAT (/T5,10I12)                                                MOT   5
      WRITE (3,100)                                                     MOT   6
      DO 110 I=1,M                                                     1MOT   7
  110 WRITE (3,120) I,(A(I,J),J=1,ID)                                  1MOT   8
  120 FORMAT (I5,5X,1P10E12.4)                                          MOT   9
      IF (ID-N) 130,150,150                                             MOT  10
  130 WRITE (3,100) (I,I=11,N)                                          MOT  11
      WRITE (3,100)                                                     MOT  12
      DO 140 I=1,M                                                     1MOT  13
  140 WRITE (3,120) I,(A(I,J),J=11,N)                                  1MOT  14
  150 RETURN                                                            MOT  15
      END                                                               MOT  16
```

```
      FUNCTION GEN (FUN,A,N)                                            GEN
C                                                                       GEN
C                             GEN = FJN(TRANS) * A * FJN                GEN
C                                                                       GEN
      DIMENSION A(20,21),FUN(20)                                        GEN
      GEN=0                                                             GEN
      DO 110 I=1,N                                                     1GEN
      DUM=0                                                            1GEN
      DO 100 J=1,N                                                     2GEN
  100 DUM=DUM+A(I,J)*FUN(J)                                            2GEN
  110 GEN=GEN+DUM*FUN(I)                                               1GEN
      RETURN                                                            GEN
      END                                                               GEN
```

```
      SUBROUTINE INVRS (B,N,A)                                          INV   1
C     A = INVERSE OF B          B UNDISTURBED                            INV   2
      DIMENSION A(20,21),D(20,21),IROW(21),ICOL(21),B(20,21)            INV   3
      DO 100 I=1,N                                                     1INV   4
      DO 100 J=1,N                                                     2INV   5
  100 A(I,J)=B(I,J)                                                    2INV   6
      M=N+1                                                             INV   7
      DO 110  I=1,N                                                    1INV   8
      IROW(I)=I                                                        1INV   9
  110 ICOL(I)=I                                                        1INV  10
      DO 260 K=1,N                                                     1INV  11
      AMAX= A(K,K)                                                     1INV  12
      DO 130 I=K,N                                                     2INV  13
      DO 130 J=K,N                                                     3INV  14
      IF(ABS( A(I,J))-ABS(AMAX))130,120,120                            3INV  15
  120 AMAX= A(I,J)                                                     3INV  16
      IC=I                                                             3INV  17
      JC=J                                                             3INV  18
  130 CONTINUE                                                         3INV  19
      KI=ICOL(K)                                                       1INV  20
      ICOL(K)=ICOL(IC)                                                 1INV  21
      ICOL(IC)=KI                                                      1INV  22
      KI=IROW(K)                                                       1INV  23
      IROW(K)=IROW(JC)                                                 1INV  24
      IROW(JC)=KI                                                      1INV  25
      IF(AMAX) 160,140,160                                             1INV  26
  140 WRITE (3,150)                                                    1INV  27
  150 FORMAT(' SOLUTION OF EXISTING MATRIX NOT POSSIBLE')              1INV  28
      GO TO 330                                                        1INV  29
  160 DO 170 J=1,N                                                     2INV  30
      E=A(K,J)                                                         2INV  31
      A(K,J)=A(IC,J)                                                   2INV  32
  170 A(IC,J)=E                                                        2INV  33
      DO 180 I=1,N                                                     2INV  34
      E=A(I,K)                                                         2INV  35
      A(I,K)=A(I,JC)                                                   2INV  36
  180 A(I,JC)=E                                                        2INV  37
      DO 210 I=1,N                                                     2INV  38
      IF(I-K) 200,190,200                                              2INV  39
  190 A(I,M)=1.                                                        2INV  40
      GO TO 210                                                        2INV  41
  200 A(I,M)=0.                                                        2INV  42
  210 CONTINUE                                                         2INV  43
      PVT=A(K,K)                                                       1INV  44
      DO 220 J=1,M                                                     2INV  45
  220 A(K,J)=A(K,J)/PVT                                                2INV  46
      DO 250 I=1,N                                                     2INV  47
      IF(I-K)230,250,230                                               2INV  48
  230 AMULT=A(I,K)                                                     2INV  49
      DO 240 J=1,M                                                     3INV  50
  240 A(I,J)=A(I,J)-AMULT*A(K,J)                                       3INV  51
  250 CONTINUE                                                         2INV  52
      DO 260 I=1,N                                                     2INV  53
  260  A(I,K)=A(I,M)                                                   2INV  54
      DO 290 I=1,N                                                     1INV  55
```

```
      DO 270 L=1,N                                                  ZINV  56
      IF(IROW(I)-L)270,280,270                                      ZINV  57
  270 CONTINUE                                                      ZINV  58
  280 DO 290 J=1,N                                                  ZINV  59
  290 D(L,J)=A(I,J)                                                 ZINV  60
      DO 320 J=1,N                                                  ZINV  61
      DO 300 L=1,N                                                  ZINV  62
      IF(ICOL(J)-L) 300,310,300                                     ZINV  63
  300 CONTINUE                                                      ZINV  64
  310 DO 320 I=1,N                                                  ZINV  65
  320 A(I,L)=D(I,J)                                                 ZINV  66
  330 RETURN                                                         INV  67
      END                                                            INV  68
```

```
      SUBROUTINE MMPY (A,B,N1,N2,N3,C)                                  MMY   1
C                                                                       MMY   2
C                             C = A * B                                 MMY   3
C                             A (N1 X N2)   B (N2 X N3)   C (N1 X N3)   MMY   4
C                                                                       MMY   5
      REAL A(20,21),B(20,21),C(20,21)                                   MMY   6
      DO 100 I=1,N1                                                    1MMY   7
      DO 100 J=1,N3                                                    2MMY   8
      C(I,J)=0.                                                        2MMY   9
      DO 100 K=1,N2                                                    3MMY  10
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                      3MMY  11
      RETURN                                                            MMY  12
      END                                                               MMY  13
```

```
      SUBROUTINE CINV (A,B,N,C,D)
C
C                        C+I*D = INVERSE OF A+I*B      I=SQRT(-1)
C
C                        B ASSUMED NON SINGULAR
C
      REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)
      CALL INVRS(B,N,C)
      CALL MMPY(C,A,N,N,N,E)
      CALL MMPY(A,E,N,N,N,C)
      DO 100 I=1,N
      DO 100 J=1,N
  100 C(I,J)=C(I,J)+B(I,J)
      CALL INVRS(C,N,D)
      CALL MMPY(E,D,N,N,N,C)
      DO 110 I=1,N
      DO 110 J=1,N
  110 D(I,J)=-D(I,J)
      RETURN
      END
```

```
      SUBROUTINE MOB (M,K,G,N,OM,ZR,ZI,YR,YI)
C
C               CALCULATES COMPLEX IMPEDANCE AND MOBILITY
C               M IS SQUARE MASS MATRIX
C               K IS SQUARE STIFFNESS MATRIX
C               G IS SCALAR STRUCTURAL DAMPING
C               OM IS FREQUENCY IN HERTZ
C               N IS ORDER
C
C               IMPEDANCE IS ZR + I*ZI     (I = SQRT(-1))
C               MOBILITY  = YR + I*YI
C
C               ALL SQUARE MATRICES ARE DIMENSIONED (20,21)
C
C        USES CINV, INVRS, MMPY
C
      REAL M(20,21),K(20,21),ZR(20,21),ZI(20,21),YR(20,21),YI(20,21)
      OMR=OM*6.283185
      CON=G/OMR
      DO 100 I=1,N
      DO 100 J=1,N
      ZR(I,J)=CON*K(I,J)
  100 ZI(I,J)=OMR*M(I,J)-K(I,J)/OMR
      CALL CINV (ZR,ZI,N,YR,YI)
      RETURN
      END
```

```
      SUBROUTINE SITER (A,PHI,FRE,J,ND,ITN,PMAX)
      REAL A(20,21),PHI(20,21),FRE(20),DUM(20)
      K=ND-J+1
      ANK=3.14159*K/(ND-1)
      AN=3.14159*J/(ND-1)
      DO 100 I=1,ND
      ANG=AN*(I-1)
      ANGK=ANK*(I-1)
  100 PHI(I,J)=(SIN(ANG)+SIN(ANGK)+1.0)/3.0
      ITN=0
      PMO=100.
  110 DO 120 I=1,ND
      DUM(I)=0.
      DO 120 L=1,ND
  120 DUM(I)=DUM(I)+A(I,L)*PHI(L,J)
      PMAX=0.
      DO 130 I=1,ND
  130 PMAX=AMAX1(PMAX,ABS(DUM(I)))
      DO 140 I=1,ND
  140 PHI(I,J)=DUM(I)/PMAX
      IF(ABS(PMAX/PMO-1.0)-.000001) 160,160,150
  150 ITN=ITN+1
      PMO=PMAX
      IF(ITN-100) 110,110,160
  160 FRE(J)=1.0/SQRT(ABS(PMAX))
      RETURN
      END
```

```
      SUBROUTINE YOUT (OMH,A,NINC,ND,NAMP)
      REAL OMH(100),A(100,20)
      J1=1
      ID=MINO(ND,10)
  100 IL=MINO(NINC,50)
      I1=1
  110 WRITE (3,120) (I,I=J1,ID)
  120 FORMAT (T5,'HERTZ'I6,9I12)
      WRITE (3,130)
  130 FORMAT (1X)
      IF(NAMP) 140,140,170
  140 DO 150 I=I1,IL
  150 WRITE(3,160) OMH(I),(A(I,J),J=J1,ID)
  160 FORMAT (1X,F9.3,1P10E12.4)
      GO TO 200
  170 DO 180 I=I1,IL
  180 WRITE(3,190) OMH(I),(A(I,J),J=J1,ID)
  190 FORMAT (1X,F9.3,10F12.2)
  200 IF(IL-NINC) 210,230,230
  210 WRITE (3,220)
  220 FORMAT ('1'//)
      I1=51
      IL=NINC
      GO TO 110
  230 IF(ID-ND) 240,250,250
  240 J1=11
      ID=ND
      WRITE (3,190)
      GO TO 100
  250 RETURN
      END
```

```
      SUBROUTINE MATAMP (OMH,A,B,NR)
C
C              CONVERTS MOBILITY, A + I*B IN VEL UNITS TO
C              AMP (IN A ) IN G'S AND PHASE ( IN B ) IN DEG
C              MATRICES ARE AT FREQUENCY  OMH IN HERTZ
C
       DIMENSION A(20,21),B(20,21)
      OM=OMH*0.01626
      DO 210 I=1,NR
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
  170 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMP (OMH,A,B,NINC,NR)
C
C                  CONVERTS A + I*B IN VELOCITY UNITS TO
C                  AMP (IN A ) IN G'S AND PHASE (IN B ) IN DEG
C                  EACH ROW IS AT A FREQUENCY OMH(I) IN HERTZ
C
      DIMENSION OMH(100),A(100,20),B(100,20)
      DO 210 I=1,NINC
      OM=OMH(I)*0.01626
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
  170 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
C     XACT   INXACT   INXACT   INXACT   INXACT   INXACT   INXACT   INXACT   INXA
      INTEGER HEADN(20),HEAD(20),HT( 7),HTN( 7),KEEP(20), INDX(100)
      REAL HZ(100),YR(20,21),YI(20,21),DPR(100,20),DPI(100,20),
     A  TR(100,20),TI(100,20)
      LOGICAL TORF
      DATA HTN/'INEXACT SIMULATED TEST DATA '/
C                                   READ INPUT CARDS AND TAPE
      READ (1,100) IC,HEADN
  100 FORMAT (I1,A3,19A4)
      READ (10) HT,HEAD,NF,ND,(HZ(I),I=1,NF)
      WRITE (3,110) HEADN,HT,HEAD,ND,(HZ(I),I=1,NF)
  110 FORMAT ('1'/T10,15(' INXACT ')//T25,A3,19A4//T10,'TAPE HEADING'/
     A T25,7A4/T25,A3,19A4/T25,I2,' DEGREES OF FREEDOM'/T25,'FREQUENCIES
     B (HZ) ON TAPE'/(T10,10F10.2))
      READ (1,120) NR,PCT,PCTB,PHE,IZ
  120 FORMAT (I10,3F10.0,30X,I10)
      IX=IZ*2+1
      WRITE (3,130) NR,PCT,PCTB,PHE,IZ
  130 FORMAT (//T10,I2,' POINTS TESTED'/T10,'MAX  RAND ERROR ='F6.3,',
     ABIAS ERROR ='F6.3,' OF ELEMENTS,  MAX RAND PHASE ERROR ='F5.2,' DE
     BG.      SEED ='I10//T10,'STATIONS USED ')
  140 READ (1,150) (KEEP(I),I=1,NR )
  150 FORMAT (16I5)
      WRITE (3,160) (KEEP(I),I=1,NR )
  160 FORMAT (T20,10I5)
  170 DO 180 I=1,NF
  180 INDX(I)=I
      READ (1,150) NFR,IP1,IP2,NROW
      IF (NFR.GT.0) READ (1,150)   (INDX(I),I=1,NFR)
      IF (NFR.EQ.0) NFR=NF
      WRITE (11) HTN,HEAD,HEADN,NFR,NR,(HZ(INDX(I)),I=1,NFR)
      WRITE (3,190) (HZ(INDX(I)),I=1,NFR)
  190 FORMAT (T10,'FREQUENCIES USED'//(T10,10F12.4))
      TORF = NROW.GT.0.AND.NROW.LE.NR
      INFR=1
      DO 290 L=1,NF
      READ (10) FREQ,((YR(I,J),YI(I,J),I=1,ND),J=1,ND)
      IF (L.NE.INDX(INFR)) GO TO 290
      IF (NR.GT.0) CALL RED (YR,YI,ND,NR,KEEP)
      IF (PCT.NE.0.OR.PCTB.NE.0.OR.PHE.NE.0) CALL ERR (YR,YI,PCT,PCTB,
     A  PHE,NR,IX)
      WRITE (11) FREQ, ((YR(I,J),YI(I,J),I=1,NR),J=1,NR)
      IF (IP1-1) 280,200,260
  200 IF (IP2.EQ.0) GO TO 220
      CALL MATAMP (FREQ,YR,YI,NR)
      WRITE (3,210) FREQ
  210 FORMAT ('1'//T40,'ACCELERATION AMPLITUDE IN G''S,PHASE IN DEG.
     A FREQ ='F10.2,'HZ'//)
      GO TO 240
  220 WRITE (3,230) FREQ
  230 FORMAT ('1'//T40,'REAL MOBILITY, IMAGINARY MOBILITY,   FREQ ='
     A F10.2,' HZ'//)
  240 CALL MOUT2(YR,NR,NR)
      WRITE (3,250)
  250 FORMAT ('1'//)
```

```
      CALL MOUT2(YI,NR,NR)
      INFR=INFR+1
      GO TO 290
  260 J=INFR
      DO 270 I=1,NR
      DPR(J,I)=YR(I,I)
      DPI(J,I)=YI(I,I)
      IF (.NOT.TORF) GO TO 270
      TR(J,I)=YR(NROW,I)
  270 TI(J,I)=YI(NROW,I)
  280 HZ(INFR)=HZ(L)
      INFR=INFR+1
      IF (INFR.GT.NFR) GO TO 300
  290 CONTINUE
  300 IF (IP1.NE.2) GO TO 390
      IF (IP2.NE.1) GO TO 320
      CALL AMP (HZ,DPR,DPI,NFR,NR)
      IF (TORF) CALL AMP (HZ,TR,TI,NFR,NR)
      WRITE (3,310)
  310 FORMAT ('1'//T30,'DRIVING POINT RESPONSE, AMP IN G''S AND PHASE IN
     A DEG'//)
      GO TO 340
  320 WRITE (3,330)
  330 FORMAT ('1'//T30,'DRIVING POINT MOBILITY, REAL AND IMAGINARY'//)
  340 CALL YOUT (HZ,DPR,NFR,NR,0)
      WRITE (3,250)
      CALL YOUT (HZ,DPI,NFR,NR,IP2)
      IF (.NOT.TORF) GO TO 390
      IF (IP2.NE.1) GO TO 360
      WRITE (3,350) NROW
  350 FORMAT ('1'//T30,'TRANSFER RESPONSE, ROW'I5,' , AMP IN G''S AND PH
     AASE IN DEG'//)
      GO TO 380
  360 WRITE (3,370) NROW
  370 FORMAT ('1'//T30,'TRANSFER MOBILITY, ROW'I5,'  REAL AND IMAG'//)
  380 CALL YOUT (HZ,TR,NFR,NR,0)
      WRITE (3,250)
      CALL YOUT (HZ,TI,NFR,NR,IP2)
  390 REWIND 10
      REWIND 11
      CALL EXIT
      END
```

```
      SUBROUTINE ERR (A,B,PCT,PCTB,PHE,N,IX)
C
C             EACH ELEMENT OF A COMPLEX MATRIX, A + I*B,IS MODIFIED TO
C             INCLUDE A SMALL PHASE ERROR, PHE (DEG),  A BIAS ERROR,
C             PCTB (RATIO) ON AMPLITUDE, AND A UNIFORM RANDOM ERROR
C             HAVING A +/- MAXIMUM OF PCT (RATIO) ON AMPLITUDE.
C             THE PHASE ERROR IS ALSO RANDOMLY DISTRIBUTED
C
C              THE RESULTING MATRIX IS SYMMETRIZED
C
C        USES RANDU
C
      DIMENSION A(20,21),B(20,21)
      IF(PCT ) 120,100,120
  100 IF(PCTB) 120,110,120
  110 IF(PHE) 120,140,120
  120 P=PHE/57.296
      DO 130 I=1,N
      DO 130 J=1,N
      CALL RANDU (IX,IY,YFL)
      IX=IY
      E=2.0*P*(YFL-0.5)
      A1=A(I,J)-E*B(I,J)
      B(I,J)=B(I,J)+E*A(I,J)
      A(I,J)=A1
      CALL RANDU (IX,IY,YFL)
      IX=IY
      E=1.0+2.0*PCT*(YFL-0.5)+PCTB
      A(I,J)=A(I,J)*E
  130 B(I,J)=B(I,J)*E
  140 N1=N-1
      DO 150 I=1,N1
      J1=I+1
      DO 150 J=J1,N
      A(I,J)=(A(I,J)+A(J,I))/2.0
      B(I,J)=(B(I,J)+B(J,I))/2.0
      B(J,I)=B(I,J)
  150 A(J,I)=A(I,J)
  160 RETURN
      END
```

```
      SUBROUTINE RANDU (IX,IY,YFL)                                      RAN   1
C                THIS SUBROUTINE IS FROM  SSP VERS. II                  RAN   2
      IY=IX*65539                                                       RAN   3
      IF(IY) 100,110,110                                                RAN   4
  100 IY=IY+2147483647+1                                                RAN   5
  110 YFL=IY                                                            RAN   6
      YFL=YFL*.4656613E-9                                               RAN   7
      RETURN                                                            RAN   8
      END                                                               RAN   9
```

```
      SUBROUTINE MATAMP (OMH,A,B,NR)
C                   CONVERTS MOBILITY, A + I*B IN VEL UNITS TO
C                   AMP (IN A ) IN G'S AND PHASE (IN B ) IN DEG
C                   MATRICES ARE AT FREQUENCY  OMH IN HERTZ
       DIMENSION A(20,21),B(20,21)
      OM=OMH*0.01626
      DO 210 I=1,NR
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
  170 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMP (OMH,A,B,NINC,NR)
C                   CONVERTS A + I*B IN VELOCITY UNITS TO
C                   AMP (IN A ) IN G'S AND PHASE ( IN B ) IN DEG
C                   EACH ROW IS AT A FREQUENCY OMH(I) IN HERTZ
      DIMENSION OMH(100),A(100,20),B(100,20)
      DO 210 I=1,NINC
      OM=OMH(I)*0.01626
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
  170 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RED (A,B,NO,NR,KEEP )                                   RED   1
      INTEGER KEEP (20)                                                  RED   2
      REAL A(20,21) , B(20,21)                                           RED   3
      DO 100 I=1,NR                                                     1RED   4
      DO 100 J=1,NR                                                     2RED   5
      A(I,J) = A(KEEP(I),KEEP(J))                                       2RED   6
  100 B(I,J) = B(KEEP(I),KEEP(J))                                       2RED   7
      RETURN                                                             RED   8
      END                                                                RED   9
```

```
      SUBROUTINE YOUT (OMH,A,NINC,ND,NAMP)
      REAL OMH(100),A(100,20)
      J1=1
      ID=MINO(ND,10)
  100 IL=MINO(NINC,50)
      I1=1
  110 WRITE (3,120) (I,I=J1,ID)
  120 FORMAT (T5,'HERTZ'I6,9I12)
      WRITE (3,130)
  130 FORMAT (1X)
      IF(NAMP) 140,140,170
  140 DO 150 I=I1,IL
  150 WRITE(3,160) OMH(I),(A(I,J),J=J1,ID)
  160 FORMAT (1X,F9.3,1P10E12.4)
      GO TO 200
  170 DO 180 I=I1,IL
  180 WRITE(3,190) OMH(I),(A(I,J),J=J1,ID)
  190 FORMAT (1X,F9.3,10F12.2)
  200 IF(IL-NINC) 210,230,230
  210 WRITE (3,220)
  220 FORMAT ('1'//)
      I1=51
      IL=NINC
      GO TO 110
  230 IF(ID-ND) 240,250,250
  240 J1=11
      ID=ND
      WRITE (3,190)
      GO TO 100
  250 RETURN
      END
```

```
      SUBROUTINE MOUT2 (A,M,N)
      REAL A(20,21)
      ID=MINO(N,10)
      WRITE (3,100) (I,I=1,ID)
  100 FORMAT (/T5,10I12)
      WRITE (3,100)
      DO 110 I=1,M
  110 WRITE (3,120) I,(A(I,J),J=1,ID)
  120 FORMAT (I5,5X,1P10E12.4)
      IF (ID-N) 130,150,150
  130 WRITE (3,100) (I,I=11,N)
      WRITE (3,100)
      DO 140 I=1,M
  140 WRITE (3,120) I,(A(I,J),J=11,N)
  150 RETURN
      END
```

```
C      IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE          IDN
       INTEGER HEADI(20),HEADN(20),HEAD(20),HTN(7),INDX(20),IT(20),       IDN
     A  IOM(20,2),ITP(20)                                                 IDN
       REAL HZ(100),YRS(20,21),YR(20,21),YI(20,21),YRSIN(20,21),DUM(20),  IDN
     A PHI(20,21),GAMMA(20,21),OM(20,2),YRSTAR(20,2),YISTAR(20,2),        IDN
     B ZRSTAR(20,2),ZISTAR(20,2),GM(20),OMEGA(20),GK(20), G(20)           IDN
       REAL DUM1(20),GAM1(20,21),FACT(20)                                 IDN
       LOGICAL EXCD                                                       IDN
  100 READ (1,110) IC,HEADI                                               IDN
       IF(IC.GT.1) CALL EXIT                                              IDN
       READ (1,110) NNOR                                                  IDN
  110 FORMAT (I1,A3,19A4)                                                 IDN
       READ (11) HTN,HEAD,HEADN,NFR,NR,(HZ(I),I=1,NFR)                    IDN
       WRITE (3,120) HEADI,HTN,HEAD,HEADN,NR,(HZ(I),I=1,NFR)              IDN
  120 FORMAT('1'//T5,12('   IDENTRE   ')//T25,A3,19A4//T10,'TAPE HEADING'/ IDN
     A T25,7A4/2(T25,A3,19A4/),T25,'ORDER OF MATRICES ='I4/T25,'FREQUENC IDN
     BIES ON TAPE'/(T10,10F10.2))                                         IDN
       READ (1,130) (INDX(I),I=1,NFR)                                     IDN
       WRITE (3,140) (HZ(INDX(I)),I=1,NFR )                               IDN
  130 FORMAT (I2I5)                                                       IDN
  140 FORMAT (//T25,'FIRST PASS FREQUENCIES'//(T10,10F10.2))              IDN
       GO TO (150,170,190,210),NNOR                                       IDN
  150 WRITE (3,160) FREQ                                                  IDN
  160 FORMAT ('     NORMALIZATION OF REAL MOBILITY BY MAX ABS(YR) ')      IDN
       GO TO 210                                                          IDN
  170 WRITE (3,180)                                                       IDN
  180 FORMAT ('      NORMALIZATION OF REAL MOBILITY BY RMS OF YR' )       IDN
       GO TO 210                                                          IDN
  190 WRITE (3,200)                                                       IDN
  200 FORMAT ('      SUMMATION OF ACCELERATION MOBILITIES ')              IDN
C                            SUMM REAL PARTS AND INVERT  (FIRST PASS)     IDN
  210 DO 220 I=1,NR                                                      1IDN
       DO 220 J=1,NR                                                     2IDN
  220 YRS(I,J)=0                                                         2IDN
       INFR=1                                                             IDN
       DO 310 L=1,NFR                                                    1IDN
       READ (11) FREQ,((YR(I,J),YI(I,J),I=1,NR),J=1,NR)                  1IDN
C      NORMALIZATION OF MOBILITY MATRICES IF NNOR=1,2                    1IDN
       IF (L.NE.INDX(INFR)) GO TO 310                                    1IDN
       GO TO (230,250,270,290),NNOR                                      1IDN
  230 CALL YRNRM (YR,NR )                                                1IDN
       WRITE (3,240) FREQ                                                1IDN
  240 FORMAT ('1'/' REAL MOB NORMALIZED ON YR (MAX)    FREQ=',F8.3,'HZ')1IDN
       GO TO 290                                                         1IDN
  250 CALL YRRMS ( YR,NR )                                               1IDN
       WRITE (3,260) FREQ                                                1IDN
  260 FORMAT ('1'/' REAL MOB NORMALIZED ON RMS OF YR   FREQ=',F8.3,'HZ')1IDN
       GO TO 290                                                         1IDN
  270 CALL YRFREQ (YR,FREQ ,NR)                                          1IDN
       WRITE (3,280)  FREQ                                               1IDN
  280 FORMAT ('1'/'  ACCELERATION MOBILITY   FREQ=',F8.3,'HZ' )          1IDN
  290 CALL MOUT2 ( YR,NR,NR )                                            1IDN
       DO 300 I=1,NR                                                     2IDN
       DO 300 J=1,NR                                                     3IDN
  300 YRS(I,J)=YRS(I,J)+YR(I,J)                                          3IDN
```

```
      INFR=INFR+1                                                       1IDN  56
      IF (INFR.GT.NFR) GO TO 320                                        1IDN  57
  310 CONTINUE                                                          1IDN  58
  320 CALL INVRS (YRS,NR,YRSIN)                                          IDN  59
      REWIND 11                                                          IDN  60
      READ (11)                                                          IDN  61
      IF (IC.EQ.0) GO TO 350                                             IDN  62
      WRITE (3,330)                                                      IDN  63
  330 FORMAT ('1'//T30,'SUM OF REAL MOBILITIES'//)                       IDN  64
      CALL MOUT2 (YRS,NR,NR)                                             IDN  65
      WRITE (3,340)                                                      IDN  66
  340 FORMAT ('1'//T30,'INVERSE OF SUM OF REAL MOB'//)                   IDN  67
      CALL MOUT2 (YRSIN,NR,NR)                                           IDN  68
C                                   ITERATE FOR PHI  (SECOND PASS)       IDN  69
  350 READ (1,130) (INDX(I),I=1,NR)                                      IDN  70
      WRITE (3,360) (HZ(INDX(I)),I=1,NR)                                 IDN  71
  360 FORMAT (//T25,'SECOND PASS FREQUENCIES'//(T10,10F10.2))            IDN  72
      INFR=1                                                             IDN  73
      DO 380 L=1,NFR                                                    1IDN  74
      READ (11) FREQ,((YR(I,J),YI(I,J),I=1,NR),J=1,NR)                  1IDN  75
      IF(L.NE.INDX(INFR)) GO TO 380                                     1IDN  76
      CALL MITER (YR,YRSIN,NR,.0001,99,DUM,VAL,ITN)                     1IDN  77
      ITP(INFR)=ITN                                                     1IDN  78
      CALL MITER (YRSIN,YR,NR,.0001,99,DUM1,VAL,ITN)                    1IDN  79
      IT(INFR)=ITN                                                      1IDN  80
      DO 370 I=1,NR                                                     2IDN  81
      GAM1(I,INFR)=DUM1(I)                                              2IDN  82
  370 PHI(I,INFR)=DUM(I)                                                2IDN  83
      INFR=INFR+1                                                       1IDN  84
      IF (INFR.GT.NR) GO TO 390                                         1IDN  85
  380 CONTINUE                                                          1IDN  86
  390 DO 420 I=1,NR                                                     1IDN  87
      SUM=0.                                                            1IDN  88
      DO 400 J=1,NR                                                     2IDN  89
  400 SUM=SUM+GAM1(J,I)*PHI(J,I)                                        2IDN  90
      DO 410 J=1,NR                                                     2IDN  91
  410 GAM1(J,I)=GAM1(J,I)/SUM                                           2IDN  92
  420 CONTINUE                                                          1IDN  93
      WRITE (3,430)                                                      IDN  94
  430 FORMAT ('1'//T40,'ITERATED PHI'//)                                 IDN  95
      CALL MOUT2 (PHI,NR,NR)                                             IDN  96
      WRITE (3,440) (ITP(I),I=1,NR)                                      IDN  97
  440 FORMAT (//T40,'ITERATIONS'//(T5,10I12))                            IDN  98
      WRITE (3,450)                                                      IDN  99
  450 FORMAT('1'//T40,'ITERATED GAMMA'//)                                IDN 100
      CALL MOUT2 (GAM1,NR,NR)                                            IDN 101
      WRITE (3,440) (IT (I),I=1,NR)                                      IDN 102
      EXCD = .FALSE.                                                     IDN 103
      DO 460 I=1,NR                                                     1IDN 104
      IF (IT(I).GT.99) EXCD=.TRUE.                                      1IDN 105
  460 CONTINUE                                                          1IDN 106
      IF (EXCD) WRITE (3,470)                                            IDN 107
  470 FORMAT (//T10,'*** WARNING - ITERATION NOT CONVERGED ***')         IDN 108
      DO 480 I=1,NR                                                     1IDN 109
      DO 480 J=1,NR                                                     2IDN 110
```

```
480 YI(I,J)=PHI(J,I)
    CALL INVRS (YI,NR,GAMMA)
    WRITE (3,490)
490 FORMAT (////T40,'GAMMA = PHI INVERSE TRANSPOSE'//)
    CALL MOUT2 (GAMMA,NR,NR)
C                          READ THIRD PASS FREQ
500 REWIND 11
    READ (11
    READ (1,130) (IOM(I,1),IOM(I,2),I=1,NR)
    WRITE (3,510) (HZ(IOM(I,1)),HZ(IOM(I,2)),I=1,NR)
510 FORMAT ('1'//T25,'THIRD PASS FREQUENCIES'//(T10,10F10.2))
C                       FORM ALL Y STAR
    I12=1
    INFR=1
    DO 550 L=1,NFR
    READ (11) FREQ,((YR(I,J),YI(I,J),I=1,NR),J=1,NR)
    IF (L.NE.IOM(INFR,I12)) GO TO 550
    OM(INFR,I12)=FREQ
    IF(I12.EQ.2) GO TO 530
    DO 520 I=1,NR
    GAMMA(I,INFR)= GAM1(I,INFR)
520 DUM(I)=GAMMA(I,INFR)
530 YRSTAR(INFR,I12)=GEN(DUM,YR,NR)
    YISTAR(INFR,I12)=GEN(DUM,YI,NR)
    IF (I12.EQ.2) GU TO 540
    I12=2
    GO TO 55C
540 I12=1
    INFR=INFR+1
    IF (INFR.GT.NR) GO TO 560
550 CONTINUE
C               FORM Z STAR
560 DO 570 L=1,NR
    DO 570 LL=1,2
    CON=YRSTAR(L,LL)**2+YISTAR(L,LL)**2
    ZRSTAR(L,LL)=YRSTAR(L,LL)/CON
570 ZISTAR(L,LL)=-YISTAR(L,LL)/CON
    WRITE (3,580 )
5o0 FORMAT ( '1',T40,'YSTAR USING ITERATED GAMMA'//)
    WRITE (3,590)
590 FORMAT (    T45,'YSTAR (MODE)'T88, 'ZSTAR (MODE)'/T3,'MODE    OM 1
   A     OM 2      REAL (OM 1)      (OM 2)   IMAG (OM 1)      (OM 2)    REAL
   B (OM 1)      (OM 2)  IMAG (OM 1)      (OM 2)'//)
    WRITE (3,600) (I,OM(I,1),YRSTAR(I,1),YISTAR(I,1),ZRSTAR(I,1),
   A  ZISTAR(I,1),OM(I,2),YRSTAR(I,2),YISTAR(I,2),ZRSTAR(I,2),
   B    ZISTAR(I,2),I=1,NR)
600 FORMAT          (I5,0PF10.2,1P4E24.4/T18,0PF10.2,1P4E24.4)
C                 IDENTIFY  GEN MASS, NAT FREQ
    DO 61C I=1,NR
    GM(I)=(OM(I,1)*ZISTAR(I,1)-OM(I,2)*ZISTAR(I,2))/(OM(I,1)**2-
   A   OM(I,2)**2)/6.283185
    OMEGA(I)=OM(I,1)*OM(I,2)*(OM(I,2)*ZISTAR(I,1)-OM(I,1)*ZISTAR(I,2))
   A  /(OM(I,1)*ZISTAR(I,1)-OM(I,2)*ZISTAR(I,2))
    GK(I)=OMEGA(I)*GM(I)*39.4784
    IF (OMEGA(I).GT.0) OMEGA(I)=SQRT(OMEGA(I))
```

```
      G(I)=OM(I,1)*ZRSTAR(I,1)/(OMEGA(I)*OMEGA(I)*GM(I)*6.283185)
  610 CONTINUE
                     WRITE (3,620) (I,GM(I),OMEGA(I),I=1,NR)
  620 FORMAT ('1'//T40,'GENERALIZED MASSES AND NATURAL FREQUENCIES'//
     A  T50,'MODE  GEN MASS',5X,'NAT FREQ'/(T50,I3,F10.4,F15.5))
      CALL REIDN (NR,GM,OMEGA,PHI,GAM1,GK,G )
C     CALL REIDN (NR,GM,OMEGA,PHI )
      REWIND 11
      GO TO 100
      END
```

```
      SUBROUTINE MITER (A,B,N,TOL,ITMAX,FUN,VAL,IT)
C                 ITERATES ON A*B FOR DOMINENT EIGENFUNCTION (FUN)
C                    AND EIGENVALUE (VAL).
C                 N IS ORDER
C                 TOL IS DECIMAL (.01 PERCENT) TOLERANCE ON VAL.
C                 ITMAX IS MAX NO OF ITERATIONS.
C                 IT IS NUMBER OF ITERATIONS PERFORMED.
C
C                 A,B ARE SQUARE OF ORDER N (DIMENSIONED (20,21) ).
C
C           USES MMPY (A,B,N1,N2,N3,C)
C
      REAL A(20,21),B(20,21),C(20,21),DUM(20),FUN(20)
      CALL MMPY (A,B,N,N,N,C)
      VALO=100.
      IT=1
      DO 100 I=1,N
  100  FUN(I)=1.0
  110 CALL MMPY (C,FUN,N,N,1,DUM)
      VAL=DUM(1)
      DO 130 I=2,N
      IF(ABS(VAL)-ABS(DUM(I))) 120,130,130
  120 VAL=DUM(I)
  130 CONTINUE
      DO 140 I=1,N
  140 FUN(I)=DUM(I)/VAL
      IF(ABS(VAL/VALO-1.0)-TOL) 160,160,150
  150 IT=IT+1
      VALO=VAL
      IF(IT-ITMAX) 110,110,160
  160 RETURN
      END
```

```
      SUBROUTINE MOUT2 (A,M,N)                                          MOT   1
      REAL A(20,21)                                                     MOT   2
      ID=MINO(N,10)                                                     MOT   3
      WRITE (3,100) (I,I=1,ID)                                          MOT   4
  100 FORMAT (/T5,10I12)                                                MOT   5
      WRITE (3,100)                                                     MOT   6
      DO 110 I=1,M                                                     1MOT   7
  110 WRITE (3,120) I,(A(I,J),J=1,ID)                                  1MOT   8
  120 FORMAT (I5,5X,1P10E12.4)                                          MOT   9
      IF (ID-N) 130,150,150                                             MOT  10
  130 WRITE (3,100) (I,I=11,N)                                          MOT  11
      WRITE (3,100)                                                     MOT  12
      DO 140 I=1,M                                                     1MOT  13
  140 WRITE (3,120) I,(A(I,J),J=11,N)                                  1MOT  14
  150 RETURN                                                            MOT  15
      END                                                               MOT  16
```

```
      FUNCTION GEN (FUN,A,N)
C                                 GEN = FJN(TRANS) * A * FJN
C
      DIMENSION A(20,21),FUN(20)
      GEN=0
      DO 110 I=1,N
      DUM=0
      DO 100 J=1,N
  100 DUM=DUM+A(I,J)*FUN(J)
  110 GEN=GEN+DUM*FUN(I)
      RETURN
      END
```

```
      SUBROUTINE INVRS (B,N,A)                                          INV
C     A = INVERSE OF B          B UNDISTURBED                           INV
C                                                                       INV
      DIMENSION A(20,21),D(20,21),IROW(21),ICOL(21),B(20, 21)           INV
      DO 100 I=1,N                                                     1INV
      DO 100 J=1,N                                                     2INV
  100 A(I,J)=B(I,J)                                                    2INV
      M=N+1                                                             INV
      DO 110  I=1,N                                                    1INV
      IROW(I)=I                                                        1INV
  110 ICOL(I)=I                                                        1INV
      DO 260 K=1,N                                                     1INV
      AMAX= A(K,K)                                                     1INV
      DO 130 I=K,N                                                     2INV
      DO 130 J=K,N                                                     3INV
      IF(ABS( A(I,J))-ABS(AMAX))130,120,120                            3INV
  120 AMAX= A(I,J)                                                     3INV
      IC=I                                                             3INV
      JC=J                                                             3INV
  130 CONTINUE                                                         3INV
      KI=ICOL(K)                                                       1INV
      ICOL(K)=ICOL(IC)                                                 1INV
      ICOL(IC)=KI                                                      1INV
      KI=IROW(K)                                                       1INV
      IROW(K)=IROW(JC)                                                 1INV
      IROW(JC)=KI                                                      1INV
      IF(AMAX) 160,140,160                                             1INV
  140 WRITE (3,150)                                                    1INV
  150 FORMAT(' SOLUTION OF EXISTING MATRIX NOT POSSIBLE')              1INV
      GO TO 330                                                        1INV
  160 DO 170 J=1,N                                                     2INV
      E=A(K,J)                                                         2INV
      A(K,J)=A(IC,J)                                                   2INV
  170 A(IC,J)=E                                                        2INV
      DO 180 I=1,N                                                     2INV
      E=A(I,K)                                                         2INV
      A(I,K)=A(I,JC)                                                   2INV
  180 A(I,JC)=E                                                        2INV
      DO 210 I=1,N                                                     2INV
      IF(I-K) 200,190,200                                              2INV
  190 A(I,M)=1.                                                        2INV
      GO TO 210                                                        2INV
  200 A(I,M)=0.                                                        2INV
  210 CONTINUE                                                         2INV
      PVT=A(K,K)                                                       1INV
      DO 220 J=1,M                                                     2INV
  220 A(K,J)=A(K,J)/PVT                                                2INV
      DO 250 I=1,N                                                     2INV
      IF(I-K)230,250,230                                               2INV
  230 AMULT=A(I,K)                                                     2INV
      DO 240 J=1,M                                                     3INV
  240 A(I,J)=A(I,J)-AMULT*A(K,J)                                       3INV
  250 CONTINUE                                                         2INV
      DO 260 I=1,N                                                     2INV
  260  A(I,K)=A(I,M)                                                   2INV
```

```
      DO 290 I=1,N                                                      1INV  56
      DO 270 L=1,N                                                      2INV  57
      IF(IROW(I)-L)270,280,270                                          2INV  58
  270 CONTINUE                                                          2INV  59
  280 DO 290 J=1,N                                                      2INV  60
  290 D(L,J)=A(I,J)                                                     2INV  61
      DO 320 J=1,N                                                      1INV  62
      DO 300 L=1,N                                                      2INV  63
      IF(ICOL(J)-L) 300,310,300                                         2INV  64
  300 CONTINUE                                                          2INV  65
  310 DO 320 I=1,N                                                      2INV  66
  320 A(I,L)=D(I,J)                                                     2INV  67
  330 RETURN                                                             INV  68
      END                                                                INV  69
```

```
      SUBROUTINE MMPY (A,B,N1,N2,N3,C)                                  MPY   1
C                                                                       MPY   2
C                          C = A * B                                    MPY   3
C                          A (N1 X N2)    B (N2 X N3)    C (N1 X N3)    MPY   4
C                                                                       MPY   5
      REAL A(20,21),B(20,21),C(20,21)                                   MPY   6
      DO 100 I=1,N1                                                    1MPY   7
      DO 100 J=1,N3                                                    2MPY   8
      C(I,J)=0.                                                        2MPY   9
      DO 100 K=1,N2                                                    3MPY  10
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                      3MPY  11
      RETURN                                                            MPY  12
      END                                                               MPY  13
```

```
      SUBROUTINE YRNRM (YR,NR )
      DIMENSION YR(20,21)
      VAL=YR(1,1)
      DO 110 I=1,NR
      DO 110 J=1,NR
      IF( ABS(VAL)-ABS(YR(I,J))) 100,110,110
  100 VAL=YR(I,J)
  110 CONTINUE
      DO 120 I=1,NR
      DO 120 J=1,NR
  120 YR(I,J)=YR(I,J)/ABS(VAL)
      RETURN
      END
      SUBROUTINE YRRMS ( YR,NR )
C     YR NORMALIZATION BY RMS OF YR
      DIMENSION YR(20,21)
      RMS=0.
      DO 130 I=1,NR
      DO 130 J=1,NR
  130 RMS=YR(I,J)*YR(I,J)+RMS
      RMS=SQRT(RMS/(NR*NR))
      DO 140 I=1,NR
      DO 140 J=1,NR
  140 YR(I,J)=YR(I,J)/RMS
      RETURN
      END
```

```
      SUBROUTINE MOB2(M,K,G,N,OM,ZR,ZI,YR,YI,D,IS)
C
C                 CALCULATES COMPLEX IMPEDANCE AND MOBILITY
C                 M IS SQUARE MASS MATRIX
C                 K IS SQUARE STIFFNESS MATRIX
C                 G IS SCALAR STRUCTURAL DAMPING
C                 D IS SQUARE DAMPING MATRIX
C                 OM IS FREQUENCY IN HERTZ
C                 N IS ORDER
C
C                 EITHER G OR D IS USED
C                 IF IS = 0  ZR = G*K/OMR
C                 IF IS = 1  ZR = D/OMR
C
C                 IMPEDANCE IS ZR + I*ZI     (I = SQRT(-1))
C                 MOBILITY  = YR + I*YI
C
C                 ALL SQUARE MATRICES ARE DIMENSIONED (20,21)
C
C        USES CINV, INVRS, MMPY
C
      REAL M(20,21),K(20,21),ZR(20,21),ZI(20,21),YR(20,21),YI(20,21)
      REAL D(20,21)
      OMR=OM*6.283185
      IF (IS.EQ.0) CON=G/OMR
      DO 110 I=1,N
      DO 110 J=1,N
      IF (IS.EQ.0) GO TO 100
      ZR(I,J)=D(I,J)/OMR
      GO TO 110
  100 ZR(I,J)=CON*K(I,J)
  110 ZI(I,J)=OMR*M(I,J)-K(I,J)/OMR
      CALL CINV (ZR,ZI,N,YR,YI)
      RETURN
      END
```

```
      SUBROUTINE CINV (A,B,N,C,D)                                       CIN   1
C                                                                       CIN   2
C                             C+I*D = INVERSE OF A+I*B      I=SQRT(-1)  CIN   3
C                                                                       CIN   4
C                             B ASSUMED NON SINGJLAR                    CIN   5
C                                                                       CIN   6
      REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)                 CIN   7
      CALL INVRS(B,N,C)                                                 CIN   8
      CALL MMPY(C,A,N,N,N,E)                                            CIN   9
      CALL MMPY(A,E,N,N,N,C)                                            CIN  10
      DO 100 I=1,N                                                     1CIN  11
      DO 100 J=1,N                                                     2CIN  12
  100 C(I,J)=C(I,J)+B(I,J)                                             2CIN  13
      CALL INVRS(C,N,D)                                                 CIN  14
      CALL MMPY(E,D,N,N,N,C)                                            CIN  15
      DO 110 I=1,N                                                     1CIN  16
      DO 110 J=1,N                                                     2CIN  17
  110 D(I,J)=-D(I,J)                                                   2CIN  18
      RETURN                                                            CIN  19
      END                                                               CIN  20
```

```
      SUBROUTINE YRFREQ (YR,FREQ,NR )                                   YRF    1
      DIMENSION YR(20,21)                                               YRF    2
      DO 100 I=1,NR                                                    1YRF    3
      DO 100 J=1,NR                                                    2YRF    4
  100 YR(I,J)=YR(I,J)*FREQ                                             2YRF    5
      RETURN                                                            YRF    6
      END                                                               YRF    7
```

```
      SUBROUTINE YOUT (OMH,A,NINC,ND,NAMP)
      REAL OMH(100),A(100,20)
      J1=1
      ID=MINO(ND,10)
  100 IL=MINO(NINC,50)
      I1=1
  110 WRITE (3,120) (I,I=J1,ID)
  120 FORMAT (T5,'HERTZ'I6,9I12)
      WRITE (3,130)
  130 FORMAT (1X)
      IF(NAMP) 140,140,170
  140 DO 150 I=I1,IL
  150 WRITE(3,160) OMH(I),(A(I,J),J=J1,ID)
  160 FORMAT (1X,F9.3,1P10E12.4)
      GO TO 200
  170 DO 180 I=I1,IL
  180 WRITE(3,190) OMH(I),(A(I,J),J=J1,ID)
  190 FORMAT (1X,F9.3,10F12.2)
  200 IF(IL-NINC) 210,230,230
  210 WRITE (3,220)
  220 FORMAT ('1'//)
      I1=51
      IL=NINC
      GO TO 110
  230 IF(ID-ND) 240,250,250
  240 J1=11
      ID=ND
      WRITE (3,190)
      GO TO 100
  250 RETURN
      END
```

```
      SUBROUTINE MATAMP (OMH,A,B,NR)
C
C                   CONVERTS MOBILITY, A + I*B IN VEL UNITS TO
C                   AMP (IN A ) IN G'S AND PHASE (IN B ) IN DEG
C                   MATRICES ARE AT FREQUENCY  OMH IN HERTZ
C
       DIMENSION A(20,21),B(20,21)
      OM=OMH*0.01626
      DO 210 I=1,NR
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
   70 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMP (OMH,A,B,NINC,NR)
C
C                 CONVERTS A + I*B IN VELOCITY UNITS TO
C                 AMP (IN A ) IN G'S AND PHASE ( IN B ) IN DEG
C                 EACH ROW IS AT A FREQUENCY OMH(I) IN HERTZ
C
      DIMENSION OMH(100),A(100,20),B(100,20)
      DO 210 I=1,NINC
      OM=OMH(I)*0.01626
      DO 210 J=1,NR
      R=A(I,J)
      C=B(I,J)
      A(I,J)=SQRT(R*R+C*C)*OM
      IF(C) 140,100,140
  100 IF(R) 110,120,130
  110 B(I,J)=270.
      GO TO 210
  120 B(I,J)=0
      GO TO 210
  130 B(I,J)=90.
      GO TO 210
  140 P=ATAN(ABS(R/C))*57.2958
      IF(C) 150,150,180
  150 IF(R) 160,160,170
  160 B(I,J)=180.+P
      GO TO 210
  170 B(I,J)=180.-P
      GO TO 210
  180 IF(R) 190,190,200
  190 B(I,J)=360.-P
      GO TO 210
  200 B(I,J)=P
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REIDN (NR,GM,OM,PHI,GAM1,GK,G )
C     IDENTIFICATION OF MASS, STIFFNESS, DAMPING MATRICES
      DIMENSION     GM(20),OM(20),PHI(20,21),AM(20,21),AK( 20,21)
      DIMENSION GAM1(20,21),GK(20),AD(20,21),G(20)
      DIMENSION C(20,21),U(20,21),CONA(20),AMG(20,21)
      DIMENSION   ZR(20,21),ZI(20,21),YR(20,21),YI(20,21),HZ(100)
      DIMENSION  DPR(100,20),DPI(100,20),TR(100,20),TI(100,20)
      LOGICAL TORF
      DO 100 I=1,NR
      DO 100 J=1,NR
      AD(I,J)=0.
      AMG(I,J)=0.
      U(I,J)=0.
  100 C(I,J)=0.
      DO 120 I=1,NR
      CONA(I)=1./(GM(I)*OM(I)*OM(I)*39.4784)
      DO 110 J=1,NR
      DO 110 K=1,NR
      CALC=PHI(K,I)*PHI(J,I)
      CAL =GAM1(K,I)*GAM1(J,I)
      AD(K,J)=CAL * G(I)*GK(I)+AD(K,J)
      AMG(K,J)=CAL*GM(I)+AMG(K,J)
      U(K,J)=CALC/GM(I) +U(K,J)
  110 C(K,J)=CALC*CONA(I)+C(K,J)
  120 CONTINUE
      CALL INVRS (C,NR,AK)
      CALL INVRS (U,NR,AM)
      WRITE (3,130)
      CALL MOUT2 (AM,NR,NR)
  130 FORMAT ('1', T50,'IDENTIFIED MASS MATRIX'//)
      WRITE (3,140)
  140 FORMAT ('1',T50, 'IDENTIFIED STIFFNESS MATRIX'//)
      CALL MOUT2 (AK,NR,NR)
      WRITE (3,150)
  150 FORMAT ('1',T50, 'IDENTIFIED DAMPING MATRIX'//)
      CALL MOUT2 (AD,NR,NR)
      SUM=0.
      WRITE (3,160)
  160 FORMAT ('  MODE NUMBER',10X,'STRUCTURAL DAMPING'/)
      DO 170 I=1,NR
      WRITE (3,180) I,G(I)
  170 SUM=SUM+G(I)
      GS=SUM/NR
  180 FORMAT (I8,F22.4)
      WRITE (3,190) GS
  190 FORMAT (//'  AVG STRUCTURAL DAMPING='F8.4)
  200 READ (1,210) NF,IP1, IP2,NROW,NN
  210 FORMAT (5I10)
      IF (NF.EQ.0) GO TO 410
      TORF=NROW.GT.0.AND.NROW.LE.NR
      READ (1,230) (HZ(I),I=1,NF)
      DO 300 L=1,NF
      OMF=HZ(L)
      CALL MOB2 (AM,AK,GS,NR,OMF,ZR,ZI,YR,YI,AD,NN)
      IF(IP1) 220,220,280
```

```
220 IF(IP2.NE.0) CALL MATAMP (HZ(L),YR,YI,NR)
    IF(IP2.NE.0) GO TO 250
    WRITE (3,240) HZ(L)
230 FORMAT (8F10.0 )
240 FORMAT ('1'T40,'REAL MOBILITY, IMAGINARY MOBILITY     FREQ ='F10.2,
   A ' HERTZ'//)
    GO TO 270
250 WRITE (3,260)  HZ(L)
260 FORMAT('1'T40,'ACCELERATION AMPLITUDE IN G''S, PHASE IN DEG.  FREQ
   A ='F10.2,' HERTZ'//)
270 CALL MOUT2 (YR,NR,NR)
    CALL MOUT2 (YI,NR,NR)
    GO TO 300
280 DO 290 I=1,NR
    DPR(L,I)=YR(I,I)
    DPI(L,I)=YI(I,I)
    IF(.NOT.TORF) GO TO 290
    TR(L,I)=YR(NROW,I)
290 TI(L,I)=YI(NROW,I)
300 CONTINUE
    IF(IP1) 410,410,310
310 IF(IP2.NE.1) GO TO 330
    CALL AMP (HZ,DPR,DPI,NF,NR)
    IF(TORF) CALL AMP (HZ,TR,TI,NF,NR)
    WRITE (3,320)
320 FORMAT ('1'T40,'DRIVING POINT RESPONSE,  AMP IN G''S AND PHASE IN
   ADEGREES'//)
    GO TO 350
330 WRITE (3,340)
340 FORMAT ('1'T40,'DRIVING POINT MOBILITY,  REAL AND IMAGINARY'//)
350 CALL YOUT (HZ,DPR,NF,NR,0)
    WRITE (3,360)
360 FORMAT ('1'//)
    CALL YOUT (HZ,DPI,NF,NR,IP2)
    IF(.NOT.TORF) GO TO 410
    IF (IP2.NE.1) GO TO 380
    WRITE (3,370) NROW
370 FORMAT ('1'T30,'TRANSFER RESPONSE, ROW 'I5,'  AMP IN G''S AND PHAS
   AE IN DEG'//)
    GO TO 400
380 WRITE (3,390) NROW
390 FORMAT ('1'T30,'TRANSFER MOBILITY, ROW 'I5,' REAL AND IMAG'//)
400 CALL YOUT (HZ,TR,NF,NR,0)
    WRITE (3,360)
    CALL YOUT (HZ,TI,NF,NR, IP2)
410 RETURN
    END
```

LIST OF FORTRAN SUBROUTINES

AMP	Converts mobility from velocity units to acceleration as amplitude (in g's) and phase angle (in degrees)
CINV	Complex inverse of complex matrix
ERR	Incorporates measurement errors into simulated measurements
GEN	Generalized function of form $f^T Af$ where f is a vector and A is a square matrix
INVRS	Inverse of a matrix
ITER	Matrix iteration for eigenvalues and eigenvectors
MITER	More general iteration on product of two matrices; used for gamma iteration
MMPX	Matrix multiplication
MØB	Calculates complex impedance and mobility
MØUT	Special output for square matrix
RANDU	Random number generator
RED	Removes rows and columns from matrix
YØUT	Special matrix output
SYM	Forms symmetric matrix from lower triangle
MØUT2	Special output for nonsquare matrix
MMPY	Matrix multiplication
SITER	Matrix iteration for eigenvalues and eigenvectors
MATAMP	Converts velocity mobility to amplitude (g's) and phase (degrees)
YRNRM	Performs normalization of mobility matrix on absolute value of largest element of mobility matrix

YRRMS	Performs normalization of mobility matrix on root mean square value of mobility matrix
MØB2	Calculates complex impedance and mobility
YRFREQ	Multiplies each velocity mobility matrix by its respective frequency to give acceleration mobility
REIDN	Identification of mass stiffness and damping matrices

SAMPLE OUTPUT

```
INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT  INXACT

           INXACT  9 POINT MODEL     20 POINT STRUCTURE    12/11/70

TAPE HEADING
           EXACT DATA SIMULATED TEST
            XACT  20 POINT UH2 8/19/70
           20 DEGREES OF FREEDOM
           FREQUENCIES (HZ) ON TAPE
     3.06      3.40      9.63     10.00     22.32     23.00     37.40     39.00     76.59     78.00
   110.52    112.00    152.90    156.00    242.00    245.80    336.28    344.00    453.00    462.34
   477.00    480.00    612.00    615.00    798.00    801.00    992.00    995.00   1226.00   1230.00
  1456.00   1460.00   1779.00   1783.00   2442.00   2447.00   3561.00   3565.00   5440.00   5445.00

 9 POINTS TESTED
MAX RAND ERROR = 0.050,   BIAS ERROR = 0.050 OF ELEMENTS,   MAX RAND PHASE ERROR = 1.00 DEG.     SEED =      206

STATIONS USED
           1    3    5    8   11   13   15   18   20
FREQUENCIES USED

     3.0600      3.4000      9.6300     10.0000     22.3200     23.0000     37.4000     39.0000     76.5900     78.0000
   110.5200    112.0000    152.9000    155.0000    242.0000    245.8000    336.2798    344.0000
```

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IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTRE   IDENTR
                    9 PT MODEL  20 PT STRUCTURE        11/13/70

   TAPE HEADING
                 INEXACT SIMULATED TEST DATA
                  XACT  20 POINT UH2 8/19/70
                 INXACT  9 POINT MODEL     20 POINT STRUCTURE    11/11/70
                 ORDER OF MATRICES =   9
                 FREQUENCIES ON TAPE
       3.06      3.40      9.63     10.00     22.32     23.00     37.40     39.00     76.59     78.00
     110.52    112.00    152.90    156.00    242.00    245.80    336.28    344.00

                 FIRST PASS FREQUENCIES

       3.06      3.40      9.63     10.00     22.32     23.00     37.40     39.00     76.59     76.00
     110.52    112.00    152.90    156.00    242.00    245.80    336.28    344.00
 SUMMATION OF ACCELERATION MOBILITIES
```

ACCELERATION MOBILITY FREQ= 3.060HZ

	1	2	3	4	5	6	7	8	9
1	1.4193E-01	1.1885E-01	8.9663E-02	6.9142E-02	4.5834E-02	3.3404E-02	1.9091E-02	-6.2770E-03	-2.4512E-02
2	1.1885E-01	9.3381E-02	7.3176E-02	5.5235E-02	3.6437E-02	2.6138E-02	1.5512E-02	-5.1528E-03	-2.0037E-02
3	8.9663E-02	7.3176E-02	5.7054E-02	4.2072E-02	2.8929E-02	1.9590E-02	1.1578E-02	-3.9627E-03	-1.6062E-02
4	6.9142E-02	5.5235E-02	4.2072E-02	3.1556E-02	2.1548E-02	1.4896E-02	8.7571E-03	-2.7923E-03	-1.1725E-02
5	4.5834E-02	3.6437E-02	2.8929E-02	2.1548E-02	1.4401E-02	1.0729E-02	6.3877E-03	-1.8185E-03	-7.8399E-03
6	3.3404E-02	2.6138E-02	1.9590E-02	1.4896E-02	1.0729E-02	7.6637E-03	4.4155E-03	-1.2225E-03	-5.5266E-03
7	1.9091E-02	1.5512E-02	1.1578E-02	8.7571E-03	6.3877E-03	4.4155E-03	2.6473E-03	-5.6178E-04	-3.1240E-03
8	-6.2770E-03	-5.1528E-03	-3.9627E-03	-2.7923E-03	-1.8185E-03	-1.2225E-03	-5.6178E-04	5.8746E-04	1.4308E-03
9	-2.4512E-02	-2.0037E-02	-1.6062E-02	-1.1725E-02	-7.8399E-03	-5.5266E-03	-3.1240E-03	1.4308E-03	4.9317E-03

ACCELERATION MOBILITY   FREQ=   3.400HZ

	1	2	3	4	5	6	7	8	9
1	4.9517E-02	3.8895E-02	2.9917E-02	2.3722E-02	1.6067E-02	1.1031E-02	6.5267E-03	-2.3836E-03	-9.2107E-03
2	3.8895E-02	3.1853E-02	2.5367E-02	1.8923E-02	1.2607E-02	9.2471E-03	5.1950E-03	-1.8770E-03	-7.3090E-03
3	2.9917E-02	2.5367E-02	1.8712E-02	1.4385E-02	1.0263E-02	6.8905E-03	4.0012E-03	-1.4133E-03	-5.5702E-03
4	2.3722E-02	1.8923E-02	1.4385E-02	1.1277E-02	7.4347E-03	5.2159E-03	3.4579E-03	-9.4974E-04	-4.0157E-03
5	1.6067E-02	1.2607E-02	1.0263E-02	7.4347E-03	5.2510E-03	3.7728E-03	2.3305E-03	-5.1095E-04	-2.5540E-03
6	1.1031E-02	9.2471E-03	6.8905E-03	5.2159E-03	3.7728E-03	2.6990E-03	1.7381E-03	-2.1509E-04	-1.7582E-03
7	6.5267E-03	5.1950E-03	4.0012E-03	3.4579E-03	2.3305E-03	1.7381E-03	1.2589E-03	1.2282E-04	-8.1639E-04
8	-2.3836E-03	-1.8770E-03	-1.4133E-03	-9.4974E-04	-5.1095E-04	-2.1509E-04	1.2282E-04	4.6168E-04	9.5873E-04
9	-9.2107E-03	-7.3090E-03	-5.5702E-03	-4.0157E-03	-2.5540E-03	-1.7582E-03	-8.1639E-04	9.5873E-04	2.2743E-03

ACCELERATION MOBILITY   FREQ=   9.630HZ

	1	2	3	4	5	6	7	8	9
1	5.3285E-01	2.2601E-01	-2.5050E-02	-2.1061E-01	-3.6045E-01	-4.2201E-01	-4.7685E-01	-4.9652E-01	-5.1415E-01
2	2.2601E-01	9.7757E-02	-1.0312E-02	-9.1107E-02	-1.5183E-01	-1.8930E-01	-2.0964E-01	-2.2340E-01	-2.2840E-01
3	-2.5050E-02	-1.0312E-02	1.2001E-03	9.4715E-03	1.6455E-02	2.0479E-02	2.2793E-02	2.3613E-02	2.4217E-02
4	-2.1061E-01	-9.1107E-02	9.4715E-03	8.2921E-02	1.4693E-01	1.7400E-01	1.9261E-01	2.0940E-01	2.1136E-01
5	-3.6045E-01	-1.5183E-01	1.6455E-02	1.4693E-01	2.4386E-01	2.9491E-01	3.3350E-01	3.5820E-01	3.4955E-01
6	-4.2201E-01	-1.8930E-01	2.0479E-02	1.7400E-01	2.9491E-01	3.4664E-01	3.8567E-01	4.1743E-01	4.2575E-01
7	-4.7685E-01	-2.0964E-01	2.2793E-02	1.9261E-01	3.3350E-01	3.8567E-01	4.5184E-01	4.6989E-01	4.8295E-01
8	-4.9652E-01	-2.2340E-01	2.3613E-02	2.0940E-01	3.5820E-01	4.1743E-01	4.6989E-01	5.0327E-01	5.3413E-01
9	-5.1415E-01	-2.2840E-01	2.4217E-02	2.1136E-01	3.4955E-01	4.2575E-01	4.8295E-01	5.3413E-01	5.1896E-01

ACCELERATION MOBILITY FREQ= 10.000HZ

	1	2	3	4	5	6	7	8	9
1	1.2720E-01	5.6919E-02	-5.8938E-03	-5.1901E-02	-8.4441E-02	-1.0464E-01	-1.1393E-01	-1.2500E-01	-1.2500E-01
2	5.6919E-02	2.3756E-02	-2.6440E-03	-2.2296E-02	-3.9799E-02	-4.6932E-02	-5.1327E-02	-5.3989E-02	-5.3889E-02
3	-5.8938E-03	-2.6440E-03	2.4358E-04	2.4210E-03	4.3186E-03	4.8332E-03	5.3029E-03	5.5197E-03	5.7280E-03
4	-5.1901E-02	-2.2296E-02	2.4210E-03	2.1823E-02	3.6137E-02	4.2776E-02	4.8703E-02	5.1147E-02	5.1756E-02
5	-8.4441E-02	-3.9799E-02	4.3186E-03	3.6137E-02	5.9979E-02	6.8708E-02	8.3266E-02	8.8020E-02	9.0634E-02
6	-1.0464E-01	-4.6932E-02	4.8332E-03	4.2776E-02	6.8708E-02	8.5224E-02	9.7038E-02	1.0570E-01	1.0709E-01
7	-1.1393E-01	-5.1327E-02	5.3029E-03	4.8703E-02	8.3266E-02	9.7038E-02	1.1205E-01	1.1335E-01	1.1389E-01
8	-1.2500E-01	-5.3989E-02	5.5197E-03	5.1147E-02	8.8020E-02	1.0570E-01	1.1335E-01	1.2078E-01	1.2732E-01
9	-1.2500E-01	-5.3889E-02	5.7280E-03	5.1756E-02	9.0634E-02	1.0709E-01	1.1389E-01	1.2732E-01	1.3303E-01

ACCELERATION MOBILITY FREQ= 22.320HZ

	1	2	3	4	5	6	7	8	9
1	9.0993E-01	2.0592E-01	-1.4744E-01	-1.3395E-01	5.0079E-02	2.4630E-01	5.1907E-01	1.3771E 00	2.2491E 00
2	2.0592E-01	4.7550E-02	-3.2826E-02	-2.9808E-02	1.0760E-02	5.7770E-02	1.1476E-01	2.9688E-01	5.4112E-01
3	-1.4744E-01	-3.2826E-02	2.3849E-02	2.1832E-02	-8.4392E-03	-4.0427E-02	-8.5937E-02	-2.2158E-01	-3.7340E-01
4	-1.3395E-01	-2.9808E-02	2.1832E-02	2.0406E-02	-6.8461E-03	-3.8214E-02	-7.6723E-02	-2.0406E-01	-3.3329E-01
5	5.0079E-02	1.0760E-02	-8.4392E-03	-6.8461E-03	3.3310E-03	1.4339E-02	2.9377E-02	7.2997E-02	1.2343E-01
6	2.4630E-01	5.7770E-02	-4.0427E-02	-3.8214E-02	1.4339E-02	7.3883E-02	1.4762E-01	3.7457E-01	6.2337E-01
7	5.1907E-01	1.1476E-01	-8.5937E-02	-7.6723E-02	2.9377E-02	1.4762E-01	3.0351E-01	7.9935E-01	1.3658E 00
8	1.3771E 00	2.9688E-01	-2.2158E-01	-2.0406E-01	7.2997E-02	3.7457E-01	7.9935E-01	2.1286E 00	3.6141E 00
9	2.2491E 00	5.4112E-01	-3.7340E-01	-3.3329E-01	1.2343E-01	6.2337E-01	1.3658E 00	3.6141E 00	6.1281E 00

ACCELERATION MOBILITY FREQ= 23.000HZ

	1	2	3	4	5	6	7	8	9
1	6.2170E-01	1.4326E-01	-1.0557E-01	-9.2963E-02	3.4671E-02	1.6881E-01	3.6934E-01	9.1123E-01	1.5577E 00
2	1.4326E-01	3.1760E-02	-2.2937E-02	-2.0792E-02	7.4432E-03	3.7747E-02	8.2834E-02	2.0373E-01	3.5535E-01
3	-1.0557E-01	-2.2937E-02	1.7353E-02	1.4192E-02	-5.7474E-03	-2.7116E-02	-5.9004E-02	-1.5041E-01	-2.5157E-01
4	-9.2963E-02	-2.0792E-02	1.4192E-02	1.3626E-02	-4.8172E-03	-2.4314E-02	-5.4437E-02	-1.4155E-01	-2.3698E-01
5	3.4671E-02	7.4432E-03	-5.7474E-03	-4.8172E-03	2.5880E-03	1.0261E-02	2.1118E-02	5.0205E-02	8.5107E-02
6	1.6881E-01	3.7747E-02	-2.7116E-02	-2.4314E-02	1.0261E-02	5.0301E-02	1.0011E-01	2.6034E-01	4.2224E-01
7	3.6934E-01	8.2834E-02	-5.9004E-02	-5.4437E-02	2.1118E-02	1.0011E-01	2.0759E-01	5.2815E-01	9.0001E-01
8	9.1123E-01	2.0373E-01	-1.5041E-01	-1.4155E-01	5.0205E-02	2.6034E-01	5.2815E-01	1.4774E 00	2.4934E 00
9	1.5577E 00	3.5535E-01	-2.5157E-01	-2.3698E-01	8.5107E-02	4.2224E-01	9.0001E-01	2.4934E 00	4.1646E 00

ACCELERATION MOBILITY FREQ= 37.400HZ

	1	2	3	4	5	6	7	8	9
1	3.2094E 00	1.1104E-01	-6.1102E-01	1.3757E-01	8.5721E-01	1.1216E 00	9.6735E-01	-5.8325E-01	-2.8250E 00
2	1.1104E-01	4.1582E-03	-2.2889E-02	4.8512E-03	3.2556E-02	4.0710E-02	3.6694E-02	-2.0334E-02	-1.0730E-01
3	-6.1102E-01	-2.2889E-02	1.2907E-01	-2.8536E-02	-1.8340E-01	-2.2588E-01	-2.0615E-01	1.1914E-01	6.0089E-01
4	1.3757E-01	4.8512E-03	-2.8536E-02	6.6101E-03	4.0766E-02	4.9245E-02	4.3704E-02	-2.7312E-02	-1.2901E-01
5	8.5721E-01	3.2556E-02	-1.8340E-01	4.0766E-02	2.6812E-01	3.1705E-01	2.8815E-01	-1.7176E-01	-8.4560E-01
6	1.1216E 00	4.0710E-02	-2.2588E-01	4.9245E-02	3.1705E-01	4.1285E-01	3.5644E-01	-2.0991E-01	-1.0908E 00
7	9.6735E-01	3.6694E-02	-2.0615E-01	4.3704E-02	2.8815E-01	3.5644E-01	3.2823E-01	-1.8334E-01	-9.8655E-01
8	-5.8325E-01	-2.0334E-02	1.1914E-01	-2.7312E-02	-1.7176E-01	-2.0991E-01	-1.8334E-01	1.1840E-01	5.8552E-01
9	-2.8250E 00	-1.0730E-01	6.0089E-01	-1.2901E-01	-8.4560E-01	-1.0908E 00	-9.8655E-01	5.8552E-01	2.8134E 00

ACCELERATION MOBILITY FREQ= 39.000HZ

	1	2	3	4	5	6	7	8	9
1	9.2367E-01	3.0306E-02	-1.7166E-01	3.8472E-02	2.4890E-01	2.9813E-01	2.7501E-01	-1.5701E-01	-8.1298E-01
2	3.0306E-02	1.7030E-03	-6.2990E-03	1.1618E-03	8.9874E-03	1.1377E-02	1.0195E-02	-5.0455E-03	-2.7880E-02
3	-1.7166E-01	-6.2990E-03	3.6168E-02	-7.4747E-03	-5.0047E-02	-6.4370E-02	-5.8873E-02	3.2891E-02	1.7067E-01
4	3.8472E-02	1.1618E-03	-7.4747E-03	1.9404E-03	1.0431E-02	1.3824E-02	1.1711E-02	-7.8808E-03	-3.3933E-02
5	2.4890E-01	8.9874E-03	-5.0047E-02	1.0431E-02	7.4925E-02	8.9583E-02	7.7492E-02	-4.6953E-02	-2.4520E-01
6	2.9813E-01	1.1377E-02	-6.4370E-02	1.3824E-02	8.9583E-02	1.1277E-01	9.7422E-02	-5.7365E-02	-2.9629E-01
7	2.7501E-01	1.0195E-02	-5.8873E-02	1.1711E-02	7.7492E-02	9.7422E-02	9.3349E-02	-4.8909E-02	-2.5950E-01
8	-1.5701E-01	-5.0455E-03	3.2891E-02	-7.8808E-03	-4.6953E-02	-5.7365E-02	-4.8909E-02	4.0996E-02	1.6138E-01
9	-8.1298E-01	-2.7880E-02	1.7067E-01	-3.3933E-02	-2.4520E-01	-2.9629E-01	-2.5950E-01	1.6138E-01	8.0850E-01

ACCELERATION MOBILITY   FREQ= 76.590HZ

	1	2	3	4	5	6	7	8	9
1	5.0301E 00	-8.9881E-01	1.3672E-01	6.2396E-01	-8.4057E-01	-2.0260E 00	-2.8206E 00	-1.5860E 00	2.8439E 00
2	-8.9881E-01	1.3930E-01	-2.4500E-02	-1.0183E-01	1.3951E-01	3.4456E-01	4.8781E-01	2.7234E-01	-4.7007E-01
3	1.3672E-01	-2.4500E-02	4.3212E-03	1.5345E-02	-2.2851E-02	-5.2457E-02	-7.2415E-02	-3.9019E-[illegible]	7.6324E-02
4	6.2396E-01	-1.0183E-01	1.5345E-02	7.3283E-02	-9.5463E-02	-2.2851E-01	-3.4314E-01	-1.90[illegible]3E-01	3.3964E-01
5	-8.4057E-01	1.3951E-01	-2.2851E-02	-9.5463E-02	1.3709E-01	3.1236E-01	4.6268E-01	2.6120E-01	-4.4836E-01
6	-2.0260E 00	3.4456E-01	-5.2457E-02	-2.2851E-01	3.1236E-01	7.6603E-01	1.1098E 00	6.1442E-01	-1.1325E 00
7	-2.8206E 00	4.8781E-01	-7.2415E-02	-3.4314E-01	4.6268E-01	1.1098E 00	1.6628E 00	8.9519E-01	-1.5710E 00
8	-1.5860E 00	2.7234E-01	-3.9019E-02	-1.9083E-01	2.6120E-01	6.1442E-01	8.9519E-01	5.0555E-01	-9.0144E-01
9	2.8439E 00	-4.7007E-01	7.6324E-02	3.3964E-01	-4.4836E-01	-1.1325E 00	-1.5710E 00	-9.0144E-01	1.6658E 00

ACCELERATION MOBILITY FREQ= 78.000HZ

	1	2	3	4	5	6	7	8	9
1	4.2086E 00	-6.8372E-01	1.1061E-01	4.5610E-01	-6.2667E-01	-1.5165E 00	-2.2052E 00	-1.2112E 00	2.2209E 00
2	-6.8372E-01	1.1669E-01	-1.8803E-02	-7.8450E-02	1.0401E-01	2.4990E-01	3.6650E-01	1.9545E-01	-3.6994E-01
3	1.1061E-01	-1.8803E-02	3.8547E-03	1.2335E-02	-1.6988E-02	-4.0954E-02	-5.5847E-02	-3.1256E-02	5.7637E-02
4	4.5610E-01	-7.8450E-02	1.2335E-02	5.7151E-02	-7.4451E-02	-1.8074E-01	-2.6330E-01	-1.4619E-01	2.5737E-01
5	-6.2667E-01	1.0401E-01	-1.6988E-02	-7.4451E-02	1.0486E-01	2.3613E-01	3.4917E-01	1.9228E-01	-3.5268E-01
6	-1.5165E 00	2.4990E-01	-4.0954E-02	-1.8074E-01	2.3613E-01	5.9319E-01	8.7111E-01	4.7586E-01	-8.8498E-01
7	-2.2052E 00	3.6650E-01	-5.5847E-02	-2.6330E-01	3.4917E-01	8.7111E-01	1.2812E 00	7.1406E-01	-1.2712E 00
8	-1.2112E 00	1.9545E-01	-3.1256E-02	-1.4619E-01	1.9228E-01	4.7586E-01	7.1406E-01	4.0458E-01	-6.8542E-01
9	2.2209E 00	-3.6994E-01	5.7637E-02	2.5737E-01	-3.5268E-01	-8.8498E-01	-1.2712E 00	-6.8542E-01	1.2928E 00

ACCELERATION MOBILITY FREQ= 110.520HZ

	1	2	3	4	5	6	7	8	9
1	4.5106E 00	-9.6081E-01	6.8829E-01	-3.6699E-01	-1.9719E-01	1.0550E 00	2.6669E 00	2.9089E 00	-3.3442E 00
2	-9.6081E-01	1.9666E-01	-1.4265E-01	7.8335E-02	4.0030E-02	-2.2732E-01	-5.6650E-01	-6.0944E-01	6.9688E-01
3	6.8829E-01	-1.4265E-01	9.9511E-02	-5.6276E-02	-2.8174E-02	1.6106E-01	3.9934E-01	4.5505E-01	-4.9583E-01
4	-3.6699E-01	7.8335E-02	-5.6276E-02	3.1951E-02	1.4953E-02	-9.0285E-02	-2.3054E-01	-2.4007E-01	2.7008E-01
5	-1.9719E-01	4.0030E-02	-2.8174E-02	1.4953E-02	1.1589E-02	-4.4569E-02	-1.1591E-01	-1.3532E-01	1.4387E-01
6	1.0550E 00	-2.2732E-01	1.6106E-01	-9.0285E-02	-4.4569E-02	2.6858E-01	6.8191E-01	7.1882E-01	-8.2026E-01
7	2.6669E 00	-5.6650E-01	3.9934E-01	-2.3054E-01	-1.1591E-01	6.8191E-01	1.6806E 00	1.8631E 00	-2.0225E 00
8	2.9089E 00	-6.0944E-01	4.5505E-01	-2.4007E-01	-1.3532E-01	7.1882E-01	1.8631E 00	2.0188E 00	-2.1999E 00
9	-3.3442E 00	6.9688E-01	-4.9583E-01	2.7008E-01	1.4387E-01	-8.2026E-01	-2.0225E 00	-2.1999E 00	2.3689E 00

ACCELERATION MOBILITY   FREQ= 112.000HZ

	1	2	3	4	5	6	7	8	9
1	3.8025E 00	-8.4398E-01	6.0902E-01	-3.2547E-01	-1.6721E-01	9.5518E-01	2.3507E 00	2.5649E 00	-2.8961E 00
2	-8.4398E-01	1.7383E-01	-1.2529E-01	6.8903E-02	3.6506E-02	-1.9648E-01	-5.0709E-01	-5.4303E-01	5.9312E-01
3	6.0902E-01	-1.2529E-01	8.7455E-02	-5.0907E-02	-2.5721E-02	1.4537E-01	3.7258E-01	3.7607E-01	-4.4245E-01
4	-3.2647E-01	6.8903E-02	-5.0907E-02	2.7255E-02	1.2828E-02	-8.2950E-02	-1.9745E-01	-2.1340E-01	2.3595E-01
5	-1.6721E-01	3.6506E-02	-2.5721E-02	1.2828E-02	9.9629E-03	-3.8399E-02	-1.0337E-01	-1.1922E-01	1.2682E-01
6	9.5518E-01	-1.9648E-01	1.4537E-01	-8.2950E-02	-3.8399E-02	2.3070E-01	5.9611E-01	6.1406E-01	-6.7230E-01
7	2.3507E 00	-5.0709E-01	3.7258E-01	-1.9745E-01	-1.0337E-01	5.9611E-01	1.4258E 00	1.5358E 00	-1.8061E 00
8	2.5649E 00	-5.4303E-01	3.7607E-01	-2.1340E-01	-1.1922E-01	6.1406E-01	1.5358E 00	1.6782E 00	-1.9021E 00
9	-2.8961E 00	5.9312E-01	-4.4245E-01	2.3595E-01	1.2682E-01	-6.7230E-01	-1.8061E 00	-1.9021E 00	2.0694E 00

ACCELERATION MOBILITY   FREQ= 152.90CHZ

	1	2	3	4	5	6	7	8	9
1	1.4439E 00	-3.0527E-01	1.5075E-01	-2.8419E-01	5.9813E-01	5.1921E-01	-3.3623E-01	-1.7748E 00	1.5790E 00
2	-3.0527E-01	6.6368E-02	-3.1588E-02	6.1162E-02	-1.2141E-01	-1.0571E-01	7.1591E-02	3.8487E-01	-3.4483E-01
3	1.5075E-01	-3.1588E-02	1.4898E-02	-3.0282E-02	5.9512E-02	5.3577E-02	-3.2973E-02	-1.7347E-01	1.6380E-01
4	-2.8419E-01	6.1162E-02	-3.0282E-02	5.6899E-02	-1.2007E-01	-1.0704E-01	7.1943E-02	3.5294E-01	-3.2588E-01
5	5.9813E-01	-1.2141E-01	5.9512E-02	-1.2007E-01	2.4263E-01	2.1379E-01	-1.4700E-01	-7.6655E-01	6.5721E-01
6	5.1921E-01	-1.0571E-01	5.3577E-02	-1.0704E-01	2.1379E-01	1.9539E-01	-1.1694E-01	-6.4345E-01	6.0804E-01
7	-3.3623E-01	7.1591E-02	-3.2973E-02	7.1943E-02	-1.4700E-01	-1.1694E-01	1.1246E-01	4.5984E-01	-4.2411E-01
8	-1.7748E 00	3.8487E-01	-1.7347E-01	3.5294E-01	-7.6655E-01	-6.4845E-01	4.5984E-01	2.3919E 00	-2.1089E 00
9	1.5790E 00	-3.4483E-01	1.6380E-01	-3.2588E-01	6.5721E-01	6.0804E-01	-4.2411E-01	-2.1089E 00	1.8778E 00

ACCELERATION MOBILITY FREQ= 156.00CHZ

	1	2	3	4	5	6	7	8	9
1	1.7613E 00	-3.5091E-01	1.6937E-01	-3.3707E-01	6.8326E-01	5.8932E-01	-3.9962E-01	-2.0208E 00	1.7927E 00
2	-3.5091E-01	7.0943E-02	-3.5169E-02	6.9292E-02	-1.4327E-01	-1.2652E-01	8.3085E-02	4.1769E-01	-3.9087E-01
3	1.6937E-01	-3.5169E-02	1.8707E-02	-3.2317E-02	6.8260E-02	6.2447E-02	-3.8364E-02	-2.0134E-01	1.8896E-01
4	-3.3707E-01	6.9292E-02	-3.2317E-02	6.3047E-02	-1.3711E-01	-1.1935E-01	8.1059E-02	4.1256E-01	-3.6202E-01
5	6.8326E-01	-1.4327E-01	6.8260E-02	-1.3711E-01	2.9087E-01	2.3847E-01	-1.7219E-01	-8.5338E-01	7.8982E-01
6	5.8932E-01	-1.2652E-01	6.2447E-02	-1.1935E-01	2.3847E-01	2.3039E-01	-1.3976E-01	-7.7002E-01	6.8901E-01
7	-3.9962E-01	8.3085E-02	-3.8364E-02	8.1059E-02	-1.7219E-01	-1.3976E-01	1.2492E-01	5.2734E-01	-4.8563E-01
8	-2.0208E 00	4.1769E-01	-2.0134E-01	4.1256E-01	-8.5338E-01	-7.7002E-01	5.2734E-01	2.5955E 00	-2.3354E 00
9	1.7927E 00	-3.9087E-01	1.8896E-01	-3.6202E-01	7.8982E-01	6.8901E-01	-4.8563E-01	-2.3354E 00	2.1613E 00

ACCELERATION MOBILITY FREQ= 242.000HZ

	1	2	3	4	5	6	7	8	9
1	8.2665E-01	-8.7173E-02	-1.6759E-01	1.4652E-01	-1.5034E-01	-1.0438E 00	-1.1317E 00	1.2022E 00	-1.0944E 00
2	-8.7173E-02	1.4048E-02	2.3114E-02	-2.0100E-02	1.9645E-02	1.4197E-01	1.5439E-01	-1.6635E-01	1.4595E-01
3	-1.6759E-01	2.3114E-02	4.7196E-02	-3.9824E-02	4.1048E-02	2.6814E-01	3.1307E-01	-3.1920E-01	2.9909E-01
4	1.4652E-01	-2.0100E-02	-3.9824E-02	3.4260E-02	-3.4849E-02	-2.3209E-01	-2.6389E-01	2.7035E-01	-2.4421E-01
5	-1.5034E-01	1.9645E-02	4.1048E-02	-3.4849E-02	3.6079E-02	2.2676E-01	2.4294E-01	-2.7506E-01	2.5012E-01
6	-1.0438E 00	1.4197E-01	2.6814E-01	-2.3209E-01	2.2676E-01	1.6179E 00	1.7889E 00	-1.9441E 00	1.7120E 00
7	-1.1317E 00	1.5439E-01	3.1307E-01	-2.6389E-01	2.4294E-01	1.7889E 00	2.1355E 00	-2.2607E 00	1.9865E 00
8	1.2022E 00	-1.6635E-01	-3.1920E-01	2.7035E-01	-2.7506E-01	-1.9441E 00	-2.2607E 00	2.3530E 00	-2.1291E 00
9	-1.0944E 00	1.4595E-01	2.9909E-01	-2.4421E-01	2.5012E-01	1.7120E 00	1.9865E 00	-2.1291E 00	1.8228E 00

ACCELERATION MOBILITY FREQ= 245.800HZ

	1	2	3	4	5	6	7	8	9
1	6.2163E-01	-6.6854E-02	-1.2893E-01	1.0655E-01	-1.0526E-01	-7.4786E-01	-8.6382E-01	9.2350E-01	-8.0851E-01
2	-6.6854E-02	8.3969E-03	1.7448E-02	-1.4507E-02	1.4135E-02	1.0602E-01	1.2296E-01	-1.2431E-01	1.1095E-01
3	-1.2893E-01	1.7448E-02	3.4205E-02	-3.0219E-02	2.9449E-02	2.0764E-01	2.2620E-01	-2.4572E-01	2.1643E-01
4	1.0655E-01	-1.4607E-02	-3.0219E-02	2.6249E-02	-2.6494E-02	-1.7506E-01	-1.8882E-01	2.0107E-01	-1.7945E-01
5	-1.0526E-01	1.4135E-02	2.9449E-02	-2.6494E-02	2.5438E-02	1.6954E-01	1.8604E-01	-2.0139E-01	1.8069E-01
6	-7.4786E-01	1.0602E-01	2.0764E-01	-1.7506E-01	1.6954E-01	1.1959E 00	1.3707E 00	-1.4702E 00	1.2703E 00
7	-8.6382E-01	1.2296E-01	2.2620E-01	-1.8882E-01	1.8604E-01	1.3707E 00	1.5105E 00	-1.6423E 00	1.4880E 00
8	9.2350E-01	-1.2431E-01	-2.4572E-01	2.0107E-01	-2.0139E-01	-1.4702E 00	-1.6423E 00	1.7145E 00	-1.5302E 00
9	-8.0851E-01	1.1095E-01	2.1643E-01	-1.7945E-01	1.8069E-01	1.2703E 00	1.4880E 00	-1.5302E 00	1.4882E 00

ACCELERATION MOBILITY   FREQ= 336.280HZ

	1	2	3	4	5	6	7	8	9
1	1.2718E 00	-2.8444E-02	-2.8397E-01	2.6913E-01	-3.9221E-01	2.2589E-01	1.2460E 00	-5.8919E-01	8.8975E-01
2	-2.8444E-02	1.9030E-03	1.5833E-02	-1.5807E-02	2.2582E-02	-1.2607E-02	-7.4099E-02	3.4405E-02	-4.9299E-02
3	-2.8397E-01	1.5833E-02	1.2582E-01	-1.1532E-01	1.7727E-01	-9.9707E-02	-5.4298E-01	2.6262E-01	-3.6921E-01
4	2.6913E-01	-1.5807E-02	-1.1532E-01	1.1712E-01	-1.7182E-01	9.5027E-02	5.3327E-01	-2.5400E-01	3.7992E-01
5	-3.9221E-01	2.2582E-02	1.7727E-01	-1.7182E-01	2.5973E-01	-1.4389E-01	-8.1980E-01	3.9409E-01	-5.8096E-01
6	2.2589E-01	-1.2607E-02	-9.9707E-02	9.5027E-02	-1.4389E-01	9.7694E-02	4.6736E-01	-2.2702E-01	3.0985E-01
7	1.2460E 00	-7.4099E-02	-5.4298E-01	5.3327E-01	-8.1980E-01	4.6736E-01	2.5672E 00	-1.2598E 00	1.7365E 00
8	-5.8919E-01	3.4405E-02	2.6262E-01	-2.5400E-01	3.9409E-01	-2.2702E-01	-1.2598E 00	6.2600E-01	-8.3557E-01
9	8.8975E-01	-4.9299E-02	-3.6921E-01	3.7992E-01	-5.8096E-01	3.0985E-01	1.7365E 00	-8.3557E-01	1.2573E 00

ACCELERATION MOBILITY FREQ= 344.000HZ

	1	2	3	4	5	6	7	8	9
1	1.4848E 00	-3.4613E-02	-3.1697E-01	3.0636E-01	-4.5154E-01	2.7267E-01	1.4385E 00	-6.8701E-01	9.6762E-01
2	-3.4613E-02	2.8336E-03	1.7505E-02	-1.8028E-02	2.5828E-02	-1.4060E-02	-8.6340E-02	3.8348E-02	-5.7468E-02
3	-3.1697E-01	1.7505E-02	1.3867E-01	-1.3308E-01	2.0320E-01	-1.1225E-01	-6.5877E-01	3.0103E-01	-4.4151E-01
4	3.0636E-01	-1.8028E-02	-1.3308E-01	1.3439E-01	-1.9488E-01	1.0824E-01	6.5069E-01	-2.9280E-01	4.4419E-01
5	-4.5154E-01	2.5828E-02	2.0320E-01	-1.9488E-01	3.0409E-01	-1.6370E-01	-9.4750E-01	4.3668E-01	-6.5066E-01
6	2.7267E-01	-1.4060E-02	-1.1225E-01	1.0824E-01	-1.6370E-01	[illegible].1086E-01	5.0545E-01	-2.5535E-01	3.5617E-01
7	1.4385E 00	-8.6340E-02	-6.5877E-01	6.5069E-01	-9.4750E-01	5.0545E-01	3.0855E 00	-1.4322E 00	2.0706E 00
8	-6.8701E-01	3.8348E-02	3.0103E-01	-2.9280E-01	4.3668E-01	-2.5535E-01	-1.4322E 00	7.2430E-01	-9.6030E-01
9	9.6762E-01	-5.7468E-02	-4.4151E-01	4.4419E-01	-6.5066E-01	3.5617E-01	2.0706E 00	-9.6030E-01	1.3548E 00

SUM OF REAL MOBILITIES

	1	2	3	4	5	6	7	8	9
1	3.1399E 01	-3.3294E 00	2.0249E-02	3.7338E-01	-8.4168E-01	-3.6424E-01	1.5105E 00	6.4850E-01	1.6468E 00
2	-3.3294E 00	1.1229E 00	-3.0346E-01	-5.4966E-02	5.4729E-02	1.0650E-01	5.6523E-02	9.0789E-02	3.1609E-01
3	2.0249E-02	-3.0346E-01	8.5829E-01	-3.9219E-01	2.9734E-01	2.8688E-01	-4.5644E-01	1.8857E-01	-5.9186E-01
4	3.7338E-01	-5.4966E-02	-3.9219E-01	8.1177E-01	-5.7577E-01	-7.7532E-01	2.7618E-02	-2.2436E-01	3.2975E-01
5	-8.4168E-01	5.4729E-02	2.9734E-01	-5.7577E-01	2.0948E 00	1.8158E 00	-2.2332E-01	-7.1829E-01	-3.3659E-01
6	-3.6424E-01	1.0650E-01	2.8688E-01	-7.7532E-01	1.8158E 00	6.3986E 00	8.3252E 00	-2.0027E 00	1.6194E 00
7	1.5105E 00	5.6523E-02	-4.5644E-01	2.7618E-02	-2.2332E-01	8.3252E 00	1.7087E 01	1.0783E 00	1.3137E 00
8	6.4850E-01	9.0789E-02	1.8857E-01	-2.2436E-01	-7.1829E-01	-2.0027E 00	1.0783E 00	1.5403E 01	-8.0701E 00
9	1.6468E 00	3.1609E-01	-5.9186E-01	3.2975E-01	-3.3659E-01	1.6194E 00	1.3137E 00	-8.0701E 00	3.1938E 01

INVERSE OF SUM OF REAL MOB

	1	2	3	4	5	6	7	8	9
1	5.0050E-02	1.6690E-01	6.0446E-02	3.2791E-02	2.7644E-03	1.2553E-02	-8.9994E-03	-2.5677E-03	-4.3368E-03
2	1.6690E-01	1.6438E 00	8.3188E-01	2.7081E-01	3.1200E-01	-3.8553E-01	1.9853E-01	-6.3128E-02	-1.3536E-02
3	6.0446E-02	8.3188E-01	2.1800E 00	4.8681E-01	9.2302E-01	-1.2304E 00	6.6695E-01	-1.3721E-01	3.4035E-02
4	3.2791E-02	2.7081E-01	4.8682E-01	2.5925E 00	-8.6144E-01	1.7123E 00	-8.5079E-01	2.0168E-01	-3.2060E-02
5	2.7644E-03	3.1200E-01	9.2302E-01	-8.6144E-01	3.4414E 00	-3.7410E 00	1.9084E 00	-3.5140E-01	8.1431E-02
6	1.2553E-02	-3.8553E-01	-1.2304E 00	1.7123E 00	-3.7410E 00	4.9497E 00	-2.5205E 00	5.0556E-01	-9.6286E-02
7	-8.9994E-03	1.985[illegible]E-01	6.6695E-01	-8.5079E-01	1.9084E 00	-2.5205E 00	1.3438E 00	-2.6197E-01	4.6083E-02
8	-2.5677E-03	-6.312[illegible]-02	-1.3721E-01	2.0168E-01	-3.5140E-01	5.0556E-01	-2.6197E-01	1.1173E-01	5.8028E-03
9	-4.3368E-03	-1.3536E-02	3.4035E-02	-3.2060E-02	8.1431E-02	-9.6286E-02	4.6083E-02	5.8028E-03	3.7941E-02

SECOND PASS FREQUENCIES

3.06	9.63	22.32	37.40	75.59	110.52	152.90	242.00	336.28

ITERATED PHI

	1	2	3	4	5	6	7	8	9
1	1.0000E 00	-9.9511E-01	3.7916E-01	1.0000E 00	1.0000E 00	1.0000E 00	-7.6925E-01	5.8849E-01	4.9849E-01
2	8.0187E-01	-4.3311E-01	8.6950E-02	3.6254E-02	-1.6946E-01	-2.0947E-01	1.6460E-01	-8.0119E-02	-2.8834E-02
3	6.2392E-01	4.5990E-02	-6.1828E-02	-2.0193E-01	2.6481E-02	1.4995E-01	-7.7471E-02	-1.3341E-01	-2.1495E-01
4	4.6670E-01	4.0241E-01	-5.6207E-02	4.4657E-02	1.2476E-01	-8.2675E-02	1.5151E-01	1.2067E-01	2.0989E-01
5	3.1394E-01	6.8590E-01	2.0501E-02	2.8495E-01	-1.6857E-01	-4.3212E-02	-3.1909E-01	-1.2413E-01	-3.2167E-01
6	2.2012E-01	8.2276E-01	1.0470E-01	3.6712E-01	-3.9328E-01	2.4179E-01	-2.7802E-01	-8.8929E-01	1.7800E-01
7	1.3013E-01	9.1414E-01	2.2319E-01	3.2192E-01	-5.8562E-01	6.0595E-01	1.9310E-01	-8.9086E-01	1.0000E 00
8	-4.3010E-02	9.8684E-01	5.8933E-01	-1.9167E-01	-3.2138E-01	6.6418E-01	1.0000E 00	1.0000E 00	-4.8853E-01
9	-1.7273E-01	1.0000E 00	1.0000E 00	-9.4865E-01	5.4342E-01	-7.3634E-01	-8.5744E-01	-8.7196E-01	6.8380E-01

ITERATIONS

4	5	5	7	9	8	6	19	5

ITERATED GAMMA

	1	2	3	4	5	6	7	8	9
1	5.3240E-02	-7.2297E-02	2.2376E-01	2.3126E-01	2.5506E-01	1.7703E-01	-9.9581E-02	3.6486E-02	2.8518E-02
2	4.6192E-01	-4.8314E-01	1.0357E 00	4.7087E-01	-5.9484E-01	-8.1112E-01	7.1786E-01	-1.3483E-01	-2.1428E-01
3	5.3650E-01	3.1730E-03	-7.8115E-01	-1.3570E 00	3.7819E-01	1.0712E 00	3.3268E-02	-1.6758E-01	-7.2743E-01
4	3.0927E-01	7.4488E-01	-8.4920E-01	7.1016E-01	2.0865E 00	-6.2510E-01	1.5099E 00	-6.8758E-01	3.5487E-01
5	2.0680E-01	2.1202E-01	-1.4160E-02	3.9465E-03	-2.0016E 00	-3.7478E-01	-1.6714E 00	1.5814E 00	-1.4604E-01
6	-1.5139E-01	2.6002E-01	3.3167E-01	1.1359E 00	2.0193E 00	4.3968E-02	1.2724E 00	-2.6085E 00	-6.5871E-01
7	8.0815E-02	-9.1226E-02	-1.3295E-01	-5.1532E-01	-1.3776E 00	2.9727E-01	-5.7806E-01	1.1001E 00	6.6482E-01
8	-1.0527E-02	1.0366E-01	5.1809E-01	2.6782E-02	1.0607E-01	1.9122E-01	3.8372E-01	-7.4285E-02	-2.2756E-01
9	-1.1670E-03	4.0643E-02	4.1899E-01	-2.0958E-01	1.0644E-01	-1.4503E-01	-1.4859E-01	-1.0119E-02	6.1432E-02

ITERATIONS

4	5	4	6	12	7	6	18	7

GAMMA = PHI INVERSE TRANSPOSE

	1	2	3	4	5	6	7	8	9
1	4.9771E-02	-7.6608E-02	2.4300E-01	2.3003E-01	2.5964E-01	1.9220E-01	-9.0876E-02	3.4732E-02	1.9339E-02
2	4.6532E-01	-4.7800E-01	9.6815E-01	3.7568E-01	-6.4113E-01	-7.2242E-01	3.3136E-01	-8.1442E-02	-3.4852E-02
3	6.0534E-01	2.9031E-02	-9.7707E-01	-1.6855E 00	-4.5391E-01	1.1964E 00	-8.6034E-01	1.5453E-01	-3.1642E-02
4	2.7163E-01	7.0803E-01	-6.3625E-01	9.2060E-01	2.3581E 00	-6.8761E-01	1.8234E 00	-7.5178E-01	-9.6828E-02
5	4.3780E-01	3.1369E-01	-5.4040E-01	-8.4248E-01	-3.4520E 00	-2.4200E-01	-3.1878E 00	2.6800E 00	1.1763E 00
6	-4.4495E-01	1.2515E-01	9.8585E-01	2.0911E 00	3.8741E 00	-1.4160E-01	3.2939E 00	-3.3677E 00	-2.2313E 00
7	2.4787E-01	-2.4425E-02	-5.0417E-01	-1.0536E 00	-2.3797E 00	3.9980E-01	-1.7018E 00	1.5347E 00	1.5049E 00
8	-5.6670E-02	9.1982E-02	5.9028E-01	1.6330E-01	3.7812E-01	1.5409E-01	5.9319E-01	-1.8539E-01	-4.1266E-01
9	9.0540E-03	4.3910E-02	4.0002E-01	-2.3290E-01	3.5203E-02	-1.3499E-01	-1.9435E-01	2.3353E-02	1.0512E-01

THIRD PASS FREQUENCIES

3.06	3.40	9.63	10.00	22.32	23.00	37.40	39.00	76.59	78.00
110.52	112.00	152.90	156.00	242.00	245.80	336.28	344.00		

YSTAR USING ITERATED GAMMA

MODE	OM 1	OM 2	YSTAR (MODE) REAL (OM 1)	(OM 2)	IMAG (OM 1)	(OM 2)	ZSTAR (MODE) REAL (OM 1)	(OM 2)	IMAG (OM 1)	(OM 2)
1	3.06	3.40	4.7737E-02	1.4685E-02	6.0094E-02	-4.0989E-02	8.1046E 00	7.7461E 00	-1.0202E 01	2.1621E 01
2	9.63	10.00	5.4232E-02	1.2909E-02	-3.5918E-02	-2.9742E-02	1.2817E 01	1.2279E 01	8.4888E 00	2.8293E 01
3	22.32	23.00	2.7338E-01	1.8165E-01	9.5339E-02	-1.5671E-01	3.2613E 00	3.1561E 00	-1.1373E 00	2.7227E 00
4	37.40	39.00	8.2268E-02	2.1644E-02	1.2287E-03	-3.7290E-02	1.2153E 01	1.1696E 01	-1.8151E-01	1.9966E 01
5	76.59	76.00	6.2450E-02	4.6673E-02	9.8975E-03	-2.7702E-02	1.5620E 01	1.5844E 01	-2.4756E 00	9.4042E 00
6	110.52	112.00	4.0707E-02	3.5185E-02	4.7320E-03	-1.4738E-02	2.4238E 01	2.4179E 01	-2.8176E 00	1.0123E 01
7	152.90	156.00	1.5707E-02	1.7655E-02	7.5959E-03	-5.8243E-03	5.1599E 01	5.1082E 01	-2.4954E 01	1.6852E 01
8	242.00	245.80	7.5210E-03	3.8804E-03	1.8826E-03	-8.4479E-04	1.2511E 02	2.4604E 02	-3.1333E 01	5.3564E 01
9	336.28	344.00	7.5837E-03	8.9333E-03	4.1601E-03	-3.2302E-03	1.0136E 02	9.8997E 01	-5.5602E 01	3.5796E 01

GENERALIZED MASSES AND NATURAL FREQUENCIES

MODE	GEN MASS	NAT FREQ
1	7.5891	3.16517
2	4.4084	9.47552
3	0.4545	22.51822
4	1.0226	37.41409
5	0.6740	76.88167
6	0.6987	110.84041
7	1.0711	154.74289
8	1.7815	243.39555
9	0.9398	340.95532

IDENTIFIED MASS MATRIX

	1	2	3	4	5	6	7	8	9
1	2.0622E-01	2.8535E-01	-1.1160E-01	1.0187E-01	-3.3755E-01	4.7988E-01	-2.7881E-01	6.1291E-02	-1.1709E-02
2	2.8535E-01	3.9931E 00	2.6458E-01	-3.6575E-01	3.7867E-01	-4.6971E-01	2.4693E-01	-6.1698E-02	3.1051E-03
3	-1.1160E-01	2.6458E-01	8.0987E 00	-3.1569E 00	8.2370E 00	-1.1269E 01	6.2004E 00	-1.3643E 00	3.2976E-01
4	1.0187E-01	-3.6575E-01	-3.1569E 00	1.2585E 01	-1.4237E 01	1.8767E 01	-1.0020E 01	2.1376E 00	-4.8045E-01
5	-3.3755E-01	3.7867E-01	8.2370E 00	-1.4237E 01	3.5797E 01	-4.2128E 01	2.2099E 01	-4.6189E 00	1.0253E 00
6	4.7988E-01	-4.6971E-01	-1.1269E 01	1.8767E 01	-4.2128E 01	5.3115E 01	-2.7970E 01	5.8978E 00	-1.2660E 00
7	-2.7881E-01	2.4693E-01	6.2004E 00	-1.0020E 01	2.2099E 01	-2.7970E 01	1.5096E 01	-3.1643E 00	6.4651E-01
8	6.1291E-02	-6.1698E-02	-1.3643E 00	2.1376E 00	-4.6188E 00	5.8978E 00	-3.1643E 00	9.5834E-01	-9.5173E-02
9	-1.1709E-02	3.1052E-03	3.2976E-01	-4.8045E-01	1.0253E 00	-1.2660E 00	6.4651E-01	-9.5173E-02	2.0270E-01

IDENTIFIED STIFFNESS MATRIX

	1	2	3	4	5	6	7	8	9
1	4.1727E 04	-1.1068E 05	1.3412E 05	-2.2876E 05	6.0884E 05	-7.9678E 05	4.1736E 05	-8.6880E 04	2.0446E 04
2	-1.1068E 05	4.0608E 05	-6.2683E 05	8.3343E 05	-1.7693E 06	2.2743E 06	-1.2000E 06	2.5555E 05	-6.1074E 04
3	1.3413E 05	-6.2682E 05	1.5403E 06	-2.6108E 06	4.5686E 06	-5.2657E 06	2.6983E 06	-5.6418E 05	1.3120E 05
4	-2.2874E 05	8.3331E 05	-2.6106E 06	7.1041E 06	-1.6450E 07	1.9660E 07	-9.8426E 06	1.9858E 06	-4.4839E 05
5	6.0877E 05	-1.7689E 06	4.5679E 06	-1.6450E 07	4.8033E 07	-6.1636E 07	3.1518E 07	-6.2949E 06	1.4202E 06
6	-7.9669E 05	2.2738E 06	-5.2648E 06	1.9659E 07	-6.1636E 07	8.2183E 07	-4.3214E 07	8.7836E 06	-1.9802E 06
7	4.1731E 05	-1.1998E 06	2.6978E 06	-9.8422E 06	3.1518E 07	-4.3214E 07	2.3485E 07	-5.0115E 06	1.1456E 06
8	-8.6870E 04	2.5550E 05	-5.6407E 05	1.9857E 06	-6.2949E 06	8.7835E 06	-5.0115E 06	1.2678E 06	-3.2644E 05
9	2.0444E 04	-6.1061E 04	1.3118E 05	-4.4837E 05	1.4202E 06	-1.9802E 06	1.1455E 06	-3.2644E 05	9.9031E 04

IDENTIFIED DAMPING MATRIX

	1	2	3	4	5	6	7	8	9
1	2.1156E 03	-8.8975E 03	-2.8250E 03	-7.5527E 03	1.6145E 04	-2.3640E 04	1.2446E 04	-2.5618E 03	7.1254E 02
2	-8.8975E 03	5.3912E 04	2.0394E 04	5.4405E 04	-8.9568E 04	1.3436E 05	-7.7911E 04	2.3160E 04	-6.4418E 03
3	-2.8250E 03	2.0394E 04	1.4470E 05	-3.9253E 04	-5.5520E 04	1.8988E 05	-1.3610E 05	4.1913E 04	-1.1143E 04
4	-7.5527E 03	5.4405E 04	-3.9253E 04	2.7207E 05	-4.2286E 05	4.1935E 05	-1.6184E 05	2.0511E 04	-2.4453E 03
5	1.6145E 04	-8.9568E 04	-5.5520E 04	-4.2286E 05	9.2250E 05	-1.0986E 06	4.6051E 05	-5.5435E 04	5.8816E 03
6	-2.3640E 04	1.3436E 05	1.[illegible]988E 05	4.1935E 05	-1.0986E 06	1.5021E 06	-6.9855E 05	9.5109E 04	-1.2117E 04
7	1.2446E 04	-7.7911E 04	-1.3610E 05	-1.6184E 05	4.6051E 05	-6.9855E 05	3.5796E 05	-5.5163E 04	9.3391E 03
8	-2.9618E 03	2.3160E 04	4.1913E 04	2.0511E 04	-5.5435E 04	9.5109E 04	-5.9163E 04	2.0273E 04	-5.9727E 03
9	7.1254E 02	-6.4418E 03	-1.1143E 04	-2.4453E 03	5.8816E 03	-1.2117E 04	9.3391E 03	-5.9727E 03	2.5683E 03

MODE NUMBER	STRUCTURAL DAMPING
1	0.0519
2	0.0496
3	0.0503
4	0.0505
5	0.0478
6	0.0497
7	0.0490
8	0.0457
9	0.0497

AVG STRUCTURAL DAMPING= 0.0493

DRIVING POINT RESPONSE, AMP IN G'S AND PHASE IN DEGREES

HERTZ	1	2	3	4	5	6	7	8	9
3.000	2.8025E-03	1.7516E-03	1.0540E-03	5.9907E-04	2.9621E-04	1.7662E-04	1.0808E-04	1.C734E-04	2.631[illegible] 04
3.100	5.2659E-03	3.3468E-03	2.0232E-03	1.1407E-03	5.3777E-04	2.9136E-04	1.4564E-04	1.1686E-04	3.2406E-04
3.200	6.4028E-03	4.1386E-03	2.5072E-03	1.4012E-03	6.2369E-04	2.9725E-04	1.0800E-04	1.1451E-04	2.2927E-04
4.000	7.1206E-04	5.5818E-04	3.5120E-04	1.7673E-04	2.9005E-05	5.0846E-05	1.0899E-04	1.9244E-04	3.1881E-04
6.000	8.9205E-05	2.2788E-04	1.7849E-04	3.8584E-05	1.4314E-04	2.5859E-04	3.5774E-04	5.4075E-04	8.8323E-04
8.000	1.3054E-03	4.8285E-05	1.4574E-04	1.4803E-04	6.3975E-04	9.7385E-04	1.2536E-03	1.6868E-03	2.3594E-03
9.000	4.6616E-03	6.7221E-04	1.3125E-04	6.9225E-04	2.1912E-03	3.2039E-03	4.0119E-03	4.9526E-03	5.8686E-03
9.400	1.1323E-02	2.0998E-03	1.3349E-04	1.8252E-03	5.3654E-03	7.7334E-03	9.5773E-03	1.1259E-02	1.1909E-02
9.500	1.1663E-02	2.2676E-03	1.4274E-04	1.9246E-03	5.5348E-03	7.9636E-03	9.8276E-03	1.1428E-02	1.1700E-02
10.000	4.9459E-03	1.1736E-03	1.4299E-04	8.9392E-04	2.4121E-03	3.4152E-03	4.1512E-03	4.5006E-03	3.7243E-03
12.000	1.1490E-03	5.0421E-04	1.2416E-04	3.2089E-04	7.3705E-04	9.8373E-04	1.1070E-03	6.9013E-04	1.0623E-03
14.000	3.2545E-04	3.9975E-04	1.0998E-04	2.3837E-04	5.0357E-04	6.2700E-04	6.1626E-04	3.0346E-04	2.9557E-03
16.000	4.4265E-04	3.4419E-04	9.2664E-05	1.9994E-04	4.0319E-04	4.5169E-04	3.2072E-04	1.2442E-03	5.4997E-03
18.000	1.4027E-03	2.9109E-04	6.7174E-05	1.6985E-04	3.3997E-04	3.0831E-04	9.3041E-05	2.8255E-03	1.0009E-02
20.000	3.3369E-03	2.0192E-04	2.6386E-05	1.2730E-04	2.8703E-04	1.3197E-04	6.8183E-04	6.6528E-03	2.1062E-02
22.000	1.2279E-02	4.5845E-04	2.6649E-04	1.8998E-04	2.2899E-04	7.6085E-04	3.8706E-03	2.7790E-02	8.1487E-02
22.400	1.6412E-02	8.4462E-04	4.2282E-04	3.6339E-04	2.3720E-04	1.2217E-03	5.5453E-03	3.8726E-02	1.1200E-01
22.500	1.6751E-02	9.2527E-04	4.4852E-04	4.0216E-04	2.4346E-04	1.2997E-03	5.7628E-03	4.0003E-02	1.1530E-01
23.000	1.2460E-02	9.4042E-04	3.9862E-04	4.2602E-04	2.5339E-04	1.1555E-03	4.6794E-03	3.1747E-02	9.0001E-02
26.000	1.3438E-03	4.9015E-04	1.2129E-04	2.3939E-04	1.5005E-04	3.0501E-04	1.1771E-03	8.1654E-03	2.0842E-02
30.000	2.7793E-03	4.0122E-04	2.9547E-05	1.9679E-04	7.2951E-05	1.8298E-04	3.7205E-04	4.6717E-03	9.2729E-03
33.000	7.5268E-03	3.6683E-04	1.9622E-04	1.7539E-04	4.1703E-04	8.0507E-04	3.7773E-04	3.6202E-03	3.8052E-03
36.000	2.5803E-02	3.2818E-04	9.4681E-04	1.4180E-04	1.9071E-03	3.2967E-03	2.4298E-03	2.6601E-03	1.7009E-02
37.000	4.5695E-02	3.3047E-04	1.8106E-03	1.6560E-04	3.6159E-03	6.0741E-03	4.6635E-03	3.0421E-03	3.8698E-02
37.400	5.1394E-02	3.5340E-04	2.0948E-03	2.0481E-04	4.1659E-03	6.9251E-03	5.3682E-03	3.7356E-03	4.7072E-02
37.500	5.1495E-02	3.5997E-04	2.1138E-03	2.1402E-04	4.2044E-03	6.9665E-03	5.4096E-03	3.8983E-03	4.8069E-02
38.000	4.4894E-02	3.7860E-04	1.9056E-03	2.3662E-04	3.7772E-03	6.1741E-03	4.8262E-03	4.2637E-03	4.5539E-02
40.000	1.9243E-02	3.5438E-04	9.2353E-04	2.0636E-04	1.8087E-03	2.8044E-03	2.1668E-03	3.5666E-03	2.5030E-02
45.000	7.3461E-03	3.0825E-04	4.7324E-04	1.7041E-04	9.0052E-04	1.2070E-03	7.1029E-04	2.7243E-03	1.4333E-02
50.000	3.6222E-03	2.6644E-04	3.6708E-04	1.4737E-04	6.7252E-04	7.2404E-04	1.5703E-04	2.2239E-03	1.1308E-02
60.000	1.9723E-03	1.4263E-04	2.8176E-04	8.5181E-05	4.3294E-04	1.5591E-04	1.5244E-03	1.2306E-03	7.9836E-0[illegible]
70.000	1.5585E-02	2.7912E-04	2.2468E-04	1.3350E-04	1.5579E-04	2.1400E-03	6.1146E-03	9.7196E-04	3.5748E-03
76.000	6.8763E-02	1.9140E-03	1.9424E-04	1.0227E-03	1.8218E-03	1.0475E-02	2.3906E-02	7.1277E-03	1.8795E-02
76.800	7.7944E-02	2.2448E-03	2.1056E-04	1.2136E-03	2.2476E-03	1.2024E-02	2.6763E-02	8.2220E-03	2.3998E-02
76.900	7.8299E-02	2.2630E-03	2.1291E-04	1.2270E-03	2.2760E-03	1.2082E-02	2.6824E-02	8.2588E-03	2.4421E-02
77.000	7.8418E-02	2.2770E-03	2.1519E-04	1.2338E-03	2.3070E-03	1.2133E-02	2.6837E-02	8.2960E-03	2.4797E-02
80.000	4.3808E-02	1.4087E-03	2.1239E-04	7.9682E-04	1.6465E-03	7.1106E-03	1.4338E-02	4.5955E-03	1.8923E-02
90.000	1.1338E-02	4.5017E-04	1.1624E-04	3.2667E-04	8.0054E-04	2.3909E-03	3.0764E-03	8.3866E-04	8.6587E-03
100.000	5.3092E-03	2.1009E-04	1.2610E-04	1.8657E-04	5.8629E-04	1.1263E-03	2.5118E-03	5.4126E-03	3.3502E-03
110.000	6.8940E-02	3.0100E-03	1.5409E-03	4.8431E-04	4.6377E-04	4.0645E-03	2.5434E-02	3.1321E-02	3.7079E-02
110.800	7.6096E-02	[illegible].3420E-03	1.7043E-03	5.7152E-04	4.9252E-04	4.7074E-03	2.7823E-02	3.3407E-02	4.1874E-02
110.900	7.6588E-02	3.3661E-03	1.7159E-03	5.7959E-04	4.9602E-04	4.7653E-03	2.7971E-02	3.3479E-02	4.2255E-02
111.000	7.6972E-02	3.3853E-03	1.7248E-03	5.8697E-04	4.9945E-04	4.8144E-03	2.8080E-02	3.3509E-02	4.2578E-02
120.000	3.2799E-02	1.5195E-03	7.6751E-04	3.5379E-04	3.6342E-04	2.6157E-03	1.1281E-02	1.0136E-02	2.0806E-02
130.000	2.0261E-02	9.8153E-04	5.0949E-04	2.2183E-04	1.4684E-04	1.6539E-03	6.9057E-03	3.3867E-03	1.3238E-02
140.000	1.3692E-02	6.9724E-04	3.9551E-04	1.0743E-04	4.8933E-04	9.6226E-04	5.0983E-03	4.6548E-03	8.0210E-03
150.000	1.2236E-02	5.8931E-04	2.9894E-04	4.8890E-04	2.5326E-03	1.5641E-03	3.7682E-03	2.5197E-02	1.4246E-02
154.000	3.0490E-02	1.4277E-03	4.4810E-04	1.0833E-03	4.7835E-03	3.7606E-03	4.3405E-03	4.7195E-02	3.5425E-02
154.700	3.3768E-02	1.5876E-03	4.9380E-04	1.1614E-03	5.0224E-03	4.0718E-03	4.6136E-03	4.9455E-02	3.8603E-02
154.900	3.4159E-02	1.6070E-03	4.9971E-04	1.1697E-03	5.0440E-03	4.1057E-03	4.6501E-03	4.9651E-02	3.8962E-02

HERTZ	1	2	3	4	5	6	7	8	9
155.000	3.4874E-02	1.6427E-03	5.1096E-04	1.1837E-03	5.0757E-03	4.1653E-03	4.7209E-03	4.9936E-02	3.9604E-02
160.000	3.2462E-02	1.5746E-03	5.3426E-04	9.3189E-04	3.5904E-03	3.3743E-03	4.7566E-03	3.4722E-02	3.3325E-02
130.000	1.9596E-02	1.0083E-03	3.7608E-04	4.4453E-04	1.4662E-03	1.2308E-03	2.7849E-03	1.2968E-02	1.8277E-02
200.000	1.6396E-02	8.8234E-04	3.0332E-04	3.3097E-04	1.0800E-03	4.1713E-04	1.1389E-03	9.2277E-03	1.4449E-02
220.000	1.3952E-02	8.0887E-04	2.1048E-04	2.3225E-04	8.4933E-04	3.0229E-03	2.6854E-03	4.0329E-03	1.0763E-02
240.000	1.3554E-02	7.5421E-04	4.1691E-04	3.5455E-04	7.2441E-04	1.9141E-02	1.9510E-02	2.2740E-02	1.7961E-02
242.000	1.6381E-02	7.9821E-04	5.4052E-04	4.6181E-04	8.4200E-04	2.2284E-02	2.2623E-02	2.8331E-02	2.3641E-02
242.100	1.6538E-02	8.0104E-04	5.4623E-04	4.6688E-04	8.4867E-04	2.2406E-02	2.2740E-02	2.8576E-02	2.3914E-02
242.200	1.6694E-02	8.0391E-04	5.5184E-04	4.7186E-04	8.5532E-04	2.2522E-02	2.2952E-02	2.8315E-02	2.4183E-02
243.000	1.7914E-02	8.2766E-04	5.9241E-04	5.0833E-04	9.0755E-04	2.3253E-02	2.3539E-02	3.0476E-02	2.6163E-02
250.000	2.0650E-02	9.0732E-04	5.8190E-04	5.1413E-04	9.9639E-04	1.7733E-02	1.7342E-02	2.7384E-02	2.7610E-02
260.000	1.8006E-02	8.6101E-04	4.0931E-04	3.6877E-04	8.0997E-04	1.0530E-02	9.3493E-03	1.8717E-02	2.1452E-02
280.000	1.5560E-02	8.1495E-04	2.3609E-04	2.1273E-04	5.2604E-04	6.4322E-03	3.5687E-03	1.3004E-02	1.6632E-02
300.000	1.3900E-02	7.9107E-04	1.0659E-04	9.7989E-05	2.4463E-04	5.0141E-03	2.7847E-03	1.0348E-02	1.3523E-02
320.000	1.1495E-02	7.7036E-04	4.9877E-04	4.8335E-04	1.0388E-03	4.1170E-03	1.2964E-02	7.6147E-03	9.5365E-03
340.000	1.9516E-02	7.7241E-04	2.5427E-03	2.4242E-03	5.6982E-03	4.4965E-03	5.5096E-02	1.6381E-02	2.9326E-02
340.600	2.0334E-02	7.7541E-04	2.5943E-03	2.4726E-03	5.8167E-03	4.6102E-03	5.6020E-02	1.7067E-02	3.0519E-02
340.700	2.0465E-02	7.7591E-04	2.6018E-03	2.4796E-03	5.8339E-03	4.6286E-03	5.6145E-02	1.7176E-02	3.0705E-02
341.000	2.0845E-02	7.7744E-04	2.6216E-03	2.4982E-03	5.8794E-03	4.6842E-03	5.6486E-02	1.7490E-02	3.1244E-02
350.000	2.3453E-02	7.9450E-04	2.1639E-03	2.0548E-03	4.8824E-03	5.1594E-03	4.4594E-02	1.8876E-02	3.2645E-02
330.000	1.8293E-02	7.7407E-04	1.0486E-03	9.8909E-04	2.3929E-03	4.2791E-03	1.9726E-02	1.3466E-02	2.2370E-02
410.000	1.6905E-02	7.6359E-04	8.1770E-04	7.6853E-04	1.8762E-03	3.9401E-03	1.4616E-02	1.1932E-02	1.9853E-02
440.000	1.6264E-02	7.5700E-04	7.2211E-04	5.7710E-04	1.6619E-03	3.7489E-03	1.2485E-02	1.1289E-02	1.8739E-02
455.000	1.6054E-02	7.5445E-04	6.9268E-04	6.4892E-04	1.5958E-03	3.6795E-03	1.1825E-02	1.1054E-02	1.8332E-02
457.900	1.6020E-02	7.5402E-04	6.8801E-04	6.4445E-04	1.5854E-03	3.6680E-03	1.1720E-02	1.1015E-02	1.8324E-02
457.900	1.6019E-02	7.5400E-04	6.8765E-04	6.4429E-04	1.5850E-03	3.6674E-03	1.1716E-02	1.1014E-02	1.8322E-02
460.000	1.5994E-02	7.5368E-04	6.8449E-04	6.4107E-04	1.5775E-03	3.6586E-03	1.1639E-02	1.0986E-02	1.8280E-02
475.000	1.5836E-02	7.5158E-04	6.6348E-04	5.2094E-04	1.5302E-03	3.6043E-03	1.1168E-02	1.0803E-02	1.8018E-02
477.000	1.5816E-02	7.5131E-04	6.6103E-04	6.1859E-04	1.5247E-03	3.5972E-03	1.1111E-02	1.0787E-02	1.7987E-02
430.000	1.5790E-02	7.5093E-04	6.5748E-04	6.1518E-04	1.5167E-03	3.5876E-03	1.1031E-02	1.0756E-02	1.7942E-02
550.000	1.5349E-02	7.4411E-04	6.0331E-04	5.6315E-04	1.3945E-03	3.4230E-03	9.8044E-03	1.0251E-02	1.7230E-02
600.000	1.5164E-02	7.4087E-04	5.8258E-04	5.4319E-04	1.3475E-03	3.3504E-03	9.3318E-03	1.0038E-02	1.6943E-02
612.000	1.5128E-02	7.4022E-04	5.7877E-04	5.3951E-04	1.3388E-03	3.3362E-03	9.2440E-03	9.9973E-03	1.6939E-02
615.000	1.5120E-02	7.4007E-04	5.7786E-04	5.3864E-04	1.3368E-03	3.3329E-03	9.2231E-03	9.9877E-03	1.6876E-02
616.000	1.5117E-02	7.4001E-04	5.7757E-04	5.3836E-04	1.3361E-03	3.3320E-03	9.2171E-03	9.9845E-03	1.6872E-02
650.000	1.5033E-02	7.3843E-04	5.6869E-04	5.2979E-04	1.3159E-03	3.2981E-03	9.0133E-03	9.8877E-03	1.6745E-02

HERTZ	1	2	3	4	5	6	7	8	9
3.000	25.15	25.81	25.97	25.57	23.57	19.86	12.56	4.08	9.86
3.100	49.69	50.38	50.53	50.07	47.73	42.79	29.75	6.43	23.49
3.200	113.36	114.18	114.36	113.88	111.17	104.20	74.24	8.54	54.00
4.000	173.08	174.91	175.23	174.30	157.51	10.41	4.49	3.31	3.77
6.000	33.21	177.00	178.77	169.23	6.46	5.05	4.59	4.18	3.81
8.000	11.32	79.29	179.05	15.55	10.22	9.67	9.35	8.52	7.11
9.000	26.25	35.09	177.31	28.95	26.28	25.82	25.47	24.18	21.31
9.400	74.77	81.23	169.46	77.37	75.22	74.77	74.40	72.79	68.64
9.500	98.69	104.82	169.67	101.27	99.22	98.78	98.38	96.69	92.08
10.000	150.45	161.53	177.74	159.14	157.48	157.06	156.59	154.39	146.37
12.000	171.38	177.04	179.03	176.19	175.24	174.73	173.90	164.92	[illegible]
14.000	158.21	178.33	178.64	177.93	177.31	176.49	174.57	29.12	6.95
16.000	20.48	178.37	177.50	178.24	177.90	176.30	169.72	10.56	6.62
18.000	11.39	177.21	173.53	177.55	177.92	173.61	80.60	10.03	8.15
20.000	13.55	168.91	126.29	172.21	177.32	147.16	22.36	14.43	13.17
22.000	44.98	80.54	60.40	92.53	170.49	62.47	50.54	48.04	46.97
22.400	75.05	101.06	88.12	107.12	165.56	89.01	80.65	78.71	77.64
22.500	84.96	109.46	97.61	114.68	165.26	98.28	90.60	88.78	87.71
23.000	127.06	146.62	138.23	149.37	170.29	138.11	133.03	131.80	130.73
25.000	146.34	176.75	169.46	177.03	171.10	165.64	169.87	171.74	170.26
30.000	16.72	178.60	78.16	178.44	47.74	34.31	159.59	175.94	171.24
33.000	15.59	178.58	23.74	177.76	22.01	19.46	40.91	176.01	145.02
36.000	34.77	175.61	39.03	167.01	38.50	36.97	40.01	165.48	52.26
37.000	66.57	169.39	70.39	148.63	69.96	68.50	70.07	147.05	78.22
37.400	89.60	168.03	93.29	149.17	92.90	91.44	92.56	147.66	99.84
37.500	95.85	168.28	99.52	150.75	99.13	97.68	98.69	149.24	105.78
38.000	122.90	171.59	126.45	160.84	126.09	124.64	125.17	159.50	131.57
40.000	160.55	177.66	163.88	175.72	163.56	162.06	160.97	174.65	166.46
45.000	171.27	178.33	175.36	178.16	174.95	172.78	165.98	177.32	175.96
50.000	169.72	177.58	177.38	177.73	176.71	172.50	104.12	177.03	177.30
60.000	30.66	168.89	178.19	170.75	175.32	69.17	12.12	171.65	176.61
70.000	19.72	33.16	177.03	36.83	102.89	21.86	17.07	42.99	151.47
76.000	66.32	71.27	163.87	72.84	79.32	68.01	[illegible]	69.34	86.29
76.800	88.39	92.92	162.10	94.47	99.89	90.10	86.68	90.28	105.01
76.900	91.41	95.90	162.26	97.43	102.76	93.13	89.70	93.16	107.69
77.000	94.38	98.82	162.39	100.37	105.55	96.10	92.68	95.99	110.32
80.000	148.93	152.29	172.53	154.04	157.12	150.87	147.26	146.35	158.71
90.000	164.80	167.05	166.65	173.17	175.12	171.11	160.23	64.42	171.82
100.000	64.86	71.93	47.97	166.93	175.59	160.98	43.87	21.85	130.70
110.000	81.55	82.77	81.77	101.27	159.18	94.63	79.55	74.33	85.80
110.800	97.04	98.20	97.39	113.21	158.67	107.90	95.24	90.17	101.66
110.900	99.06	100.22	99.43	114.85	158.78	109.68	97.28	92.22	103.61
111.000	101.07	102.22	101.45	116.49	158.94	111.46	99.32	94.27	105.55
120.000	167.33	168.16	168.34	170.10	169.20	169.29	166.71	160.09	168.72
130.000	172.91	173.72	174.63	170.15	132.35	171.87	173.41	148.20	172.39
140.000	169.02	170.54	174.50	137.13	36.54	160.36	174.15	43.42	161.22
150.000	108.91	116.84	155.24	62.05	48.36	77.54	164.64	48.26	78.80
154.000	112.06	115.42	139.13	94.05	86.39	99.32	152.01	85.84	101.24
154.700	119.09	121.97	141.73	103.50	96.48	[illegible]	152.73	95.89	109.88
154.800	120.16	122.99	142.23	104.88	97.94	109.13	152.94	97.35	111.15

HERTZ	1	2	3	4	5	6	7	8	9
155.000	122.34	125.05	143.31	107.64	100.87	111.67	153.44	100.26	113.71
160.000	160.44	161.71	167.35	153.86	149.88	154.25	168.74	149.05	156.95
180.000	176.86	177.44	177.16	175.21	174.45	167.36	172.69	173.09	176.14
200.000	177.69	178.62	176.49	176.03	175.56	77.90	151.77	173.20	176.83
220.000	176.44	178.48	168.89	170.64	175.44	26.27	34.49	150.91	171.66
240.000	144.06	168.51	98.00	105.26	147.46	67.12	56.68	81.74	106.84
242.000	142.71	166.49	107.61	112.74	145.32	83.38	82.53	95.73	114.50
242.100	142.82	166.45	108.24	113.28	145.37	84.29	83.41	96.53	115.04
242.200	142.95	166.41	108.87	113.83	145.44	85.19	84.30	97.34	115.60
243.000	144.50	166.36	114.23	118.57	146.48	92.61	91.56	104.02	120.42
250.000	166.24	174.11	153.56	155.13	164.98	142.89	140.43	149.93	157.56
260.000	175.25	178.07	167.76	168.21	171.96	164.16	159.19	168.19	171.82
280.000	177.72	179.25	166.21	165.73	167.97	173.46	153.97	174.81	176.04
300.000	177.06	179.44	111.06	105.21	117.49	175.58	56.08	174.76	174.54
320.000	170.26	179.23	42.81	41.96	44.04	174.13	35.01	164.60	157.83
340.000	135.32	176.38	93.55	93.30	93.96	155.31	90.58	125.49	118.11
340.600	136.83	176.35	97.44	97.20	97.85	155.65	94.53	127.64	120.78
340.700	137.11	176.34	98.10	97.86	98.50	155.73	95.20	128.02	121.24
341.000	137.97	176.34	100.07	99.83	100.47	155.98	97.20	129.17	122.64
350.000	165.34	176.19	145.16	144.99	145.45	169.28	142.97	159.64	157.20
380.000	177.63	179.66	172.91	172.83	173.05	177.87	171.73	176.67	176.44
410.000	178.98	179.80	176.73	176.68	176.81	178.80	175.95	178.45	178.47
440.000	179.39	179.85	178.04	178.00	178.09	179.15	177.47	179.02	179.09
455.000	179.50	179.87	178.40	178.36	178.44	179.26	177.90	179.18	179.26
457.800	179.51	179.87	178.45	178.42	178.49	179.28	177.96	179.20	179.28
457.900	179.51	179.87	178.45	178.42	178.49	179.28	177.96	179.20	179.28
460.000	179.52	179.87	178.49	178.46	178.53	179.29	178.01	179.22	179.30
475.000	179.59	179.88	178.73	178.70	178.76	179.37	178.30	179.33	179.41
477.000	179.60	179.88	178.76	178.73	178.79	179.38	178.33	179.34	179.42
480.000	179.61	179.89	178.80	178.77	178.82	179.39	178.38	179.36	179.44
550.000	179.77	179.92	179.33	179.31	179.35	179.61	179.07	179.61	179.68
600.000	179.85	179.94	179.51	179.50	179.52	179.69	179.31	179.70	179.76
612.000	179.84	179.94	179.55	179.53	179.55	179.71	179.35	179.72	179.78
615.000	179.84	179.94	179.55	179.54	179.56	179.71	179.36	179.72	179.78
616.000	179.84	179.94	179.55	179.54	179.56	179.71	179.36	179.72	179.78
650.000	179.86	179.95	179.63	179.61	179.63	179.75	179.46	179.76	179.82

TRANSFER RESPONSE, ROW 5 AMP IN G'S AND PHASE IN DEG

HERTZ	1	2	3	4	5	6	7	8	9
3.000	8.1187E-04	6.6303E-04	5.3022E-04	4.1245E-04	2.9621E-04	2.2312E-04	1.5093E-04	1.7764E-05	1.0919E-04
3.100	1.6012E-03	1.2939E-03	1.0188E-03	7.7551E-04	5.3777E-04	3.9062E-04	2.4814E-04	5.0174E-05	2.5116E-04
3.200	2.0410E-03	1.6311E-03	1.2628E-03	9.3802E-04	6.2369E-04	4.3215E-04	2.5071E-04	1.1501E-04	3.6898E-04
4.000	3.6265E-04	2.6489E-04	1.7574E-04	9.8715E-05	2.9005E-05	1.6185E-05	4.7116E-05	9.8283E-05	1.3112E-04
6.000	3.9358E-04	2.3132E-04	6.4274E-05	3.8821E-05	1.4314E-04	1.9616E-04	2.3361E-04	2.7001E-04	2.8113E-04
8.000	1.0567E-03	5.1617E-04	4.2748E-05	3.3302E-04	6.3975E-04	7.8957E-04	8.8956E-04	9.6713E-04	9.7586E-04
9.000	3.2737E-03	1.4768E-03	8.2376E-05	1.2470E-03	2.1912E-03	2.6483E-03	2.9526E-03	3.1888E-03	3.2219E-03
9.400	7.8021E-03	3.4164E-03	3.4365E-04	3.1333E-03	5.3654E-03	6.4442E-03	7.1643E-03	7.7338E-03	7.8318E-03
9.500	8.0068E-03	3.4811E-03	3.8907E-04	3.2513E-03	5.5348E-03	6.6377E-03	7.3741E-03	7.9587E-03	8.0633E-03
10.000	3.4146E-03	1.4318E-03	2.4099E-04	1.4562E-03	2.4121E-03	2.8728E-03	3.1825E-03	3.4386E-03	3.4992E-03
12.000	1.0264E-03	3.7851E-04	1.4002E-04	4.8349E-04	7.3705E-04	8.5872E-04	9.4456E-04	1.0343E-03	1.0793E-03
14.000	7.4977E-04	2.4518E-04	1.3274E-04	3.5422E-04	5.0357E-04	5.7523E-04	6.3046E-04	7.0813E-04	7.6601E-04
16.000	6.8206E-04	1.9931E-04	1.3774E-04	3.0565E-04	4.0319E-04	4.4970E-04	4.9089E-04	5.6951E-04	6.4260E-04
18.000	7.0037E-04	1.8342E-04	1.5079E-04	2.8363E-04	3.3997E-04	3.6498E-04	3.9334E-04	4.6889E-04	5.5049E-04
20.000	8.0690E-04	1.9016E-04	1.7629E-04	2.7858E-04	2.8703E-04	2.8236E-04	2.8684E-04	3.2655E-04	3.8109E-04
22.000	1.2317E-03	2.6819E-04	2.5066E-04	3.1313E-04	2.2899E-04	2.0623E-04	2.9555E-04	7.2572E-04	1.2274E-03
22.400	1.2548E-03	2.7146E-04	2.4771E-04	2.9441E-04	2.3720E-04	3.2355E-04	5.5406E-04	1.3478E-03	2.2487E-03
22.500	1.1881E-03	2.5650E-04	2.3372E-04	2.7675E-04	2.4346E-04	3.5840E-04	6.1434E-04	1.4802E-03	2.4639E-03
23.000	6.2419E-04	1.2798E-04	1.3878E-04	1.8501E-04	2.5339E-04	3.9774E-04	6.5177E-04	1.5189E-03	2.5185E-03
26.000	7.7262E-04	1.1139E-04	1.9688E-04	1.9641E-04	1.5005E-04	1.7467E-04	2.9295E-04	8.5239E-04	1.5563E-03
30.000	1.4102E-03	1.4792E-04	3.3173E-04	1.7852E-04	7.2951E-05	1.2302E-04	6.9362E-05	8.3850E-04	1.9391E-03
33.000	2.5692E-03	1.9527E-04	5.7178E-04	1.3381E-04	4.1703E-04	5.7816E-04	4.1572E-04	1.0219E-03	3.0023E-03
36.000	7.5529E-03	3.7108E-04	1.5783E-03	1.8713E-04	1.9071E-03	2.4963E-03	2.2179E-03	1.9002E-03	7.6616E-03
37.000	1.3062E-02	5.3131E-04	2.6672E-03	5.3036E-04	3.6159E-03	4.6777E-03	4.0730E-03	2.7512E-03	1.2649E-02
37.400	1.4593E-02	5.4735E-04	2.9520E-03	6.8820E-04	4.1659E-03	5.3682E-03	4.7130E-03	2.8638E-03	1.3904E-02
37.500	1.4600E-02	5.3378E-04	2.9461E-03	7.1724E-04	4.2044E-03	5.4108E-03	4.7608E-03	2.7968E-03	1.3820E-02
38.000	1.2658E-02	4.1155E-04	2.5233E-03	7.2939E-04	3.7772E-03	4.8374E-03	4.2956E-03	2.1928E-03	1.1707E-02
40.000	5.4257E-03	9.3972E-05	1.0271E-03	4.8805E-04	1.8087E-03	2.2767E-03	2.0772E-03	5.8344E-04	4.5625E-03
45.000	2.3697E-03	5.3370E-05	3.8669E-04	3.6817E-04	9.0052E-04	1.0919E-03	1.0276E-03	4.1419E-05	1.5469E-03
50.000	1.8381E-03	9.8141E-05	2.4696E-04	3.5695E-04	6.7252E-04	7.7999E-04	7.3452E-04	1.4522E-04	8.5445E-04
60.000	1.9035E-03	2.0085E-04	1.4278E-04	4.0763E-04	4.3294E-04	3.9818E-04	3.1474E-04	1.1810E-04	2.2431E-04
70.000	3.7268E-03	5.4541E-04	6.4704E-05	6.5393E-04	1.5579E-04	5.4756E-04	9.5967E-04	4.5565E-04	1.0120E-03
76.000	1.1922E-02	1.9875E-03	2.8317E-04	1.5932E-03	1.8218E-03	4.3385E-03	6.4800E-03	3.4941E-03	6.0531E-03
76.800	1.3116E-02	2.2192E-03	3.6565E-04	1.6701E-03	2.2476E-03	5.1662E-03	7.6643E-03	4.1967E-03	7.1003E-03
76.900	1.3124E-02	2.2240E-03	3.7182E-04	1.6622E-03	2.2760E-03	5.2147E-03	7.7321E-03	4.2406E-03	7.1569E-03
77.000	1.3101E-02	2.2254E-03	3.7996E-04	1.6466E-03	2.3070E-03	5.2590E-03	7.7898E-03	4.2822E-03	7.2007E-03
80.000	6.7084E-03	1.2000E-03	3.0183E-04	6.7592E-04	1.6465E-03	3.4525E-03	5.0526E-03	2.9367E-03	4.5698E-03
90.000	1.7128E-03	3.5247E-04	1.6398E-04	4.0554E-05	8.0054E-04	1.5729E-03	2.3777E-03	1.6498E-03	2.1349E-03
100.000	7.3521E-04	1.7287E-04	1.0351E-04	9.8857E-05	5.8629E-04	1.2858E-03	2.1583E-03	1.7978E-03	2.0698E-03
110.000	2.9392E-03	6.1794E-04	4.3965E-04	2.7122E-04	4.6377E-04	1.3176E-03	2.7851E-03	2.8883E-03	3.1268E-03
110.800	3.4397E-03	7.3005E-04	5.2166E-04	3.3961E-04	4.9252E-04	1.1793E-03	2.5493E-03	2.6969E-03	2.9155E-03
110.900	3.4867E-03	7.4084E-04	5.2964E-04	3.4726E-04	4.9602E-04	1.1563E-03	2.5057E-03	2.6574E-03	2.8719E-03
111.000	3.5280E-03	7.5044E-04	5.3678E-04	3.5443E-04	4.9945E-04	1.1324E-03	2.4591E-03	2.6142E-03	2.8244E-03
120.000	2.3495E-03	5.2866E-04	3.8778E-04	3.9508E-04	3.6342E-04	5.0340E-04	1.0123E-03	1.1847E-03	1.0966E-03
130.000	2.3050E-03	5.2557E-04	3.7480E-04	4.7962E-04	1.4684E-04	3.8133E-04	1.3348E-03	2.0788E-03	1.8817E-03
140.000	3.2362E-03	7.2879E-04	4.7273E-04	7.1249E-04	4.8933E-04	2.5124E-04	1.6815E-03	3.6844E-03	3.2459E-03
150.000	7.5601E-03	1.6514E-03	8.9545E-04	1.5861E-03	2.5326E-03	2.0071E-03	2.7197E-03	9.6054E-03	8.2911E-03
154.000	1.1757E-02	2.5269E-03	1.2347E-03	2.3575E-03	4.7835E-03	4.1554E-03	3.3220E-03	1.5284E-02	1.3114E-02
154.700	1.1980E-02	2.5683E-03	1.2294E-03	2.3808E-03	5.0224E-03	4.4183E-03	3.2215E-03	1.5628E-02	1.3396E-02
154.800	1.1987E-02	2.5674E-03	1.2252E-03	2.3777E-03	5.0440E-03	4.4453E-03	3.1972E-03	1.5634E-02	1.3400E-02

HERTZ	1	2	3	4	5	6	7	8	9
155.000	1.1963E-02	2.5600E-03	1.2143E-03	2.3664E-03	5.0757E-03	4.4882E-03	3.1448E-03	1.5619E-02	1.3383E-02
160.000	7.0822E-03	1.4803E-03	5.8826E-04	1.2900E-03	3.5904E-03	3.4020E-03	1.1762E-03	9.3660E-03	7.9821E-03
180.000	1.9986E-03	3.7208E-04	3.9560E-05	1.9163E-04	1.4662E-03	1.5712E-03	5.5339E-04	2.4774E-03	2.1041E-03
200.000	1.4252E-03	2.3222E-04	1.3096E-04	3.9167E-05	1.0800E-03	1.1720E-03	7.1894E-04	1.3663E-03	1.1913E-03
220.000	1.4749E-03	2.1125E-04	2.4616E-04	1.6481E-04	8.4933E-04	7.6330E-04	6.1939E-04	7.0135E-04	6.6040E-04
240.000	2.5031E-03	3.3031E-04	5.6285E-04	4.8748E-04	7.2441E-04	2.4709E-03	2.3901E-03	2.7891E-03	2.4027E-03
242.000	2.5182E-03	3.3191E-04	5.7710E-04	5.0932E-04	8.4200E-04	3.1909E-03	3.1523E-03	3.5417E-03	3.0883E-03
242.100	2.5129E-03	3.3121E-04	5.7640E-04	5.0918E-04	8.4867E-04	3.2239E-03	3.1883E-03	3.5754E-03	3.1196E-03
242.200	2.5071E-03	3.3044E-04	5.7557E-04	5.0891E-04	8.5532E-04	3.2563E-03	3.2237E-03	3.6086E-03	3.1505E-03
243.000	2.4352E-03	3.2103E-04	5.6300E-04	5.0152E-04	9.0755E-04	3.4901E-03	3.4843E-03	3.8441E-03	3.3717E-03
250.000	1.0251E-03	1.3613E-04	2.5140E-04	2.3651E-04	9.9639E-04	3.4488E-03	3.7012E-03	3.6544E-03	3.3253E-03
260.000	4.6457E-04	4.0104E-05	1.4693E-04	1.2236E-04	8.0997E-04	2.5997E-03	3.0916E-03	2.6957E-03	2.5666E-03
280.000	9.3572E-04	7.1216E-05	3.2903E-04	2.9743E-04	5.2304E-04	2.1379E-03	3.1690E-03	2.2882E-03	2.3745E-03
300.000	1.5848E-03	1.1181E-04	6.0557E-04	5.7165E-04	2.4463E-04	2.1410E-03	4.1478E-03	2.5831E-03	2.9341E-03
320.000	3.1482E-03	2.0147E-04	1.2865E-03	1.2412E-03	1.0888E-03	2.5507E-03	7.1094E-03	3.9063E-03	4.8397E-03
340.000	8.8619E-03	5.1442E-04	3.8169E-03	3.7270E-03	5.6982E-03	3.6098E-03	1.7896E-02	8.7974E-03	1.2221E-02
340.600	8.9272E-03	5.1673E-04	3.8504E-03	3.7607E-03	5.8167E-03	3.5423E-03	1.7972E-02	8.8071E-03	1.2274E-02
340.700	8.9339E-03	5.1688E-04	3.8542E-03	3.7646E-03	5.8339E-03	3.5292E-03	1.7977E-02	8.8044E-03	1.2277E-02
341.000	8.9458E-03	5.1681E-04	3.8619E-03	3.7726E-03	5.8794E-03	3.4890E-03	1.7976E-02	8.7891E-03	1.2276E-02
350.000	6.2381E-03	3.4547E-04	2.7464E-03	2.6919E-03	4.8824E-03	1.5357E-03	1.1986E-02	5.5929E-03	8.2057E-03
380.000	2.0583E-03	9.9906E-05	9.5513E-04	9.4194E-04	2.3929E-03	6.2630E-04	3.4469E-03	1.3870E-03	2.4221E-03
410.000	1.2600E-03	5.4386E-05	6.0750E-04	5.9997E-04	1.8762E-03	8.4948E-04	1.8566E-03	6.4699E-04	1.3665E-03
440.000	9.3919E-04	3.6542E-05	4.6594E-04	4.5974E-04	1.6619E-03	9.3815E-04	1.2237E-03	3.6775E-04	9.5664E-04
455.000	8.4226E-04	3.1259E-05	4.2274E-04	4.1671E-04	1.5958E-03	9.6323E-04	1.0334E-03	2.8717E-04	8.3573E-04
457.800	8.2698E-04	3.0432E-05	4.1590E-04	4.0984E-04	1.5854E-03	9.6716E-04	1.0034E-03	2.7467E-04	8.1682E-04
457.900	8.2644E-04	3.0403E-05	4.1566E-04	4.0965E-04	1.5850E-03	9.6721E-04	1.0024E-03	2.7422E-04	8.1617E-04
460.000	8.1549E-04	2.9812E-05	4.1076E-04	4.0476E-04	1.5775E-03	9.7000E-04	9.8093E-04	2.6528E-04	8.0263E-04
475.000	7.4723E-04	2.6154E-05	3.8009E-04	3.7408E-04	1.5302E-03	9.8672E-04	8.4717E-04	2.1058E-04	7.1901E-04
477.000	7.3935E-04	2.5734E-05	3.7653E-04	3.7052E-04	1.5247E-03	9.8858E-04	8.3178E-04	2.0431E-04	7.0941E-04
480.000	7.2768E-04	2.5125E-05	3.7136E-04	3.6533E-04	1.5167E-03	9.9131E-04	8.0934E-04	1.9529E-04	6.9549E-04
550.000	5.5634E-04	1.6235E-05	2.9310E-04	2.8640E-04	1.3945E-03	1.0286E-03	4.7414E-04	6.6816E-05	4.9194E-04
600.000	4.9263E-04	1.3078E-05	2.6348E-04	2.5622E-04	1.3475E-03	1.0403E-03	3.4968E-04	2.3699E-05	4.1917E-04
612.000	4.8108E-04	1.2516E-05	2.5805E-04	2.5068E-04	1.3388E-03	1.0423E-03	3.2716E-04	1.6575E-05	4.0616E-04
615.000	4.7834E-04	1.2384E-05	2.567[illegible]E-04	2.4937E-04	1.3368E-03	1.0428E-03	3.2172E-04	1.5001E-05	4.0309E-04
616.000	4.7745E-04	1.2341E-05	2.5636E-04	2.4894E-04	1.3361E-03	1.0429E-03	3.2001E-04	1.4502E-05	4.0209E-04
650.000	4.5075E-04	1.1062E-05	2.4377E-04	2.3603E-04	1.3159E-03	1.0472E-03	2.6780E-04	9.0514E-06	3.7238E-04

HERTZ	1	2	3	4	5	6	7	8	9
3.000	27.10	26.64	25.97	25.04	23.57	22.09	19.61	309.57	214.63
3.100	51.61	51.13	50.43	49.42	47.73	45.90	42.52	275.15	238.13
3.200	115.59	115.09	114.33	113.20	111.17	108.77	103.65	319.03	301.47
4.000	177.05	176.41	175.18	172.41	157.51	36.71	9.83	2.44	0.42
6.000	182.59	181.73	178.25	15.01	6.46	5.54	5.07	4.53	4.21
8.000	188.71	187.76	169.16	11.54	10.22	9.93	9.77	9.64	9.62
9.000	205.34	204.37	53.44	27.17	26.28	26.06	25.95	25.92	25.98
9.400	254.48	253.52	89.11	76.02	75.22	75.03	74.94	74.94	75.03
9.500	278.51	277.53	111.75	100.01	99.22	99.02	98.93	98.94	99.03
10.000	336.99	336.00	165.43	158.20	157.48	157.30	157.23	157.27	157.41
12.000	355.49	354.45	178.73	175.86	175.24	175.08	175.04	175.21	175.47
14.000	358.27	357.14	180.05	178.00	177.31	177.10	177.07	177.36	177.75
16.000	359.71	358.46	180.75	178.83	177.90	177.57	177.50	177.86	178.34
18.000	1.00	359.62	181.50	179.39	177.92	177.24	176.93	177.07	177.45
20.000	3.33	1.93	183.11	180.40	177.32	175.00	172.84	169.35	167.20
22.000	23.04	23.07	198.80	192.06	170.49	135.71	103.41	81.58	76.97
22.400	44.00	45.66	215.65	204.67	165.56	128.98	111.40	101.87	100.01
22.500	50.48	52.80	220.48	207.86	165.26	132.48	117.95	110.22	108.77
23.000	62.79	70.97	222.03	204.05	170.29	155.61	149.76	147.33	147.16
26.000	6.05	3.84	184.92	180.08	171.10	168.30	171.98	177.90	179.65
30.000	7.07	3.06	186.32	177.81	47.74	35.48	108.65	181.45	184.43
33.000	11.86	6.11	190.95	170.21	22.01	20.54	25.68	185.31	189.56
36.000	33.26	24.22	212.11	79.33	38.50	37.84	39.42	204.56	210.90
37.000	65.59	54.64	244.36	90.81	69.96	69.39	70.59	235.77	243.17
37.400	88.83	76.95	267.56	110.02	92.90	92.35	93.44	258.54	266.39
37.500	95.13	82.93	273.85	115.39	99.13	98.59	99.65	264.62	272.67
38.000	122.43	108.68	301.10	139.29	126.09	125.57	126.51	291.13	299.92
40.000	161.03	134.61	339.50	170.89	163.56	163.11	163.71	324.86	338.29
45.000	174.47	14.26	352.31	178.78	174.95	174.46	174.64	224.83	350.79
50.000	178.04	5.94	354.99	180.16	176.71	175.94	175.74	184.50	352.55
60.000	182.65	6.35	355.78	182.[illegible]	175.32	170.64	165.13	168.54	335.83
70.000	192.65	45.11	336.30	189.[illegible]	102.89	38.11	30.51	32.91	206.74
76.000	242.12	64.16	268.98	235.7[illegible]	79.32	73.36	72.22	74.32	250.97
76.800	264.52	86.52	287.22	257.[illegible]	99.89	95.02	94.14	96.22	273.04
76.900	267.58	89.57	289.89	260.58	102.76	98.01	97.15	99.23	276.06
77.000	270.59	92.58	292.40	263.50	105.55	100.93	100.10	102.17	279.03
80.000	326.32	148.17	340.27	316.38	157.12	154.76	154.55	156.62	333.93
90.000	348.94	170.53	355.28	264.42	175.12	175.44	176.32	178.57	356.73
100.000	340.38	163.81	345.74	184.38	175.59	179.16	181.29	184.04	2.76
110.000	276.43	99.28	279.86	125.37	159.18	209.96	218.98	223.83	44.16
110.800	290.01	112.37	292.99	133.32	158.67	218.52	229.76	235.20	56.01
110.900	291.83	114.13	294.76	134.55	158.78	219.52	231.08	236.61	57.49
111.000	293.64	115.89	296.53	135.81	158.94	220.45	232.36	237.97	58.93
120.000	354.67	175.22	355.25	180.59	169.20	181.34	190.37	195.78	16.33
130.000	3.15	183.12	2.01	185.38	132.35	167.86	183.94	189.55	9.04
140.000	11.00	190.56	7.90	191.00	36.54	82.73	186.29	194.15	13.67
150.000	36.50	215.70	30.66	214.47	48.36	[illegible]5.12	205.01	218.00	37.65
154.000	76.85	255.91	69.45	253.95	86.39	90.75	240.64	257.83	77.54
154.700	87.28	266.32	79.58	264.23	96.48	100.57	249.99	268.18	87.90
154.800	88.79	267.83	81.04	265.72	97.94	101.99	251.33	269.68	89.40

HERTZ	1	2	3	4	5	6	7	8	9
155.000	91.81	270.84	83.96	268.69	100.87	104.85	254.07	272.68	92.40
160.000	142.80	321.67	132.08	318.46	149.88	152.53	291.81	323.09	142.89
180.000	172.21	350.59	72.44	338.79	174.45	175.20	188.26	350.26	170.40
200.000	178.10	356.19	10.86	245.72	176.56	175.85	180.25	352.38	173.02
220.000	183.83	2.05	10.23	196.74	175.44	165.15	169.11	338.74	161.91
240.000	221.16	41.40	44.23	227.83	147.46	91.59	95.50	266.53	89.53
242.000	235.24	56.15	57.82	241.58	145.32	103.68	107.46	279.76	102.32
242.100	236.02	56.97	58.57	242.34	145.37	104.40	108.18	280.54	103.08
242.200	236.80	57.79	59.32	243.11	145.44	105.13	108.91	281.32	103.84
243.000	243.12	64.45	65.40	249.31	146.48	111.24	114.98	287.77	110.17
250.000	275.05	102.82	93.48	280.28	164.98	154.41	158.06	332.82	154.62
260.000	222.78	63.14	39.16	227.01	171.96	171.51	175.35	351.22	172.84
280.000	191.43	10.93	13.09	194.45	167.97	178.49	183.23	359.98	181.71
300.000	192.17	10.23	13.77	194.41	117.49	181.69	188.23	5.37	187.36
320.000	201.74	19.51	23.08	203.44	44.04	188.67	198.90	16.25	198.53
340.000	262.96	80.67	94.07	264.29	93.96	240.17	260.48	77.92	260.51
340.600	267.01	84.72	88.12	268.33	97.85	243.66	264.54	81.98	264.58
340.700	267.69	85.40	88.80	269.01	98.50	244.25	265.22	82.67	265.26
341.000	269.73	87.44	90.84	271.05	100.47	246.01	267.26	84.71	267.31
350.000	316.67	134.38	157.69	317.86	145.45	279.20	314.27	131.73	314.48
380.000	347.59	165.36	168.38	348.44	173.05	193.47	345.23	162.64	345.90
410.000	352.87	170.74	173.[illegible]	353.51	176.81	183.16	350.46	167.66	351.50
440.000	355.04	173.02	175.55	355.52	178.09	181.35	352.53	169.35	353.88
455.000	355.70	173.74	176.17	356.12	178.44	180.97	353.14	169.69	354.64
457.800	355.81	173.86	176.27	356.22	178.49	180.92	353.24	169.73	354.76
457.900	355.81	173.86	176.27	356.22	178.49	180.92	353.25	169.73	354.77
460.000	355.89	173.95	176.34	356.29	178.53	180.88	353.32	169.75	354.86
475.000	356.37	174.48	176.78	356.72	178.76	180.67	353.74	169.81	355.41
477.000	356.43	174.55	176.83	356.78	178.79	180.64	353.79	169.80	355.47
480.000	356.51	174.64	176.[illegible]	356.85	178.82	180.61	353.86	169.78	355.57
550.000	357.76	176.15	178.04	357.96	179.35	180.24	354.88	166.73	357.07
600.000	358.25	176.80	178.47	358.39	179.52	180.14	355.20	155.05	357.69
612.000	358.35	176.92	178.55	358.47	179.55	180.13	355.25	146.55	357.80
615.000	358.37	176.95	178.57	358.49	179.56	180.13	355.26	143.43	357.83
616.000	358.37	176.96	178.58	358.50	179.56	180.12	355.27	142.28	357.84
650.000	358.59	177.27	178.77	358.69	179.63	180.09	355.36	51.06	358.11